Chapter:01

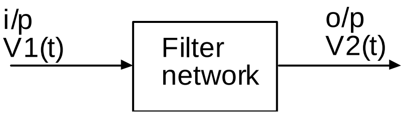
Filter: Filter can be considered can be considered as frequency selective networks. A filter is required to separate an unwanted signal from a mixture of wanted and unwanted signals.

The filter specification are generally given in terms of cutoff frequencies, pass band (P.B) and stop band (s.b) regions. P. B is the frequency band of wanted signal and S.B is the frequency band of unwanted signal. An ideal filter should pass the wanted signal with no attenuation and provide infinite attenuation.

Depending upon the components used, filters can be classified as:

1. passive filters: Filters which are the compotnet such as R,L,C are the passive filters. The Gains of such filters are always less than or equal to unity (i.e GS1). It is to be noted the L and C are filter components, but R is not.
2. Active filters: The filters which use the components such as transistors, op-amp etc are the active filters. The Gains of such filters are always greater than or equal to unity. ($G \geq 1$)

Gain and Attenuation:



Let us consider the filters network with i/p $V_1(t)$ having power P1 and o/p $V_2(t)$ having power p2 as shown in fig1. Then the transfer function is given by $T(s) = V_2(s)/V_1(s)$

Where , $V_1(s)$ and $V_2(s)$ are the Laplace Transform of $V_1(t)$.

Also, $T(s) = T(jw) = \frac{v_2(jw)}{v_1(jw)}$

Then the voltage gain in db is given by ,

$Av = 20\log_{10} |T(jw)|$ dB …………….(1)

Or in term of power , the power gain is given by,

$Ap = 10 \log_{10} \left|\frac{p_1}{p_2}\right|$

Now, the voltage attenuation is given by ,

$\alpha = 1/Av$

$\alpha = -20\log |T(jw)|$ dB…………….(2)

From equation 1 and 2 ,we can write,

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$|T(jw)| = 10^{0.05Av}$……………….(3)

$|T(jw)| = 10^{-0.05\ \alpha}$…………….(4)

Types of filters: (According to the function)

Filters are classified according to the functions they are to perform. The pattern of PB and SB that give rise to the most common filters as defined below:

1. **Low pass filters: (LPF):** A LPF characteristics is one in which the PB extend from ω = 0 to ω = ω_c where ω_c is know as cut off frequency.

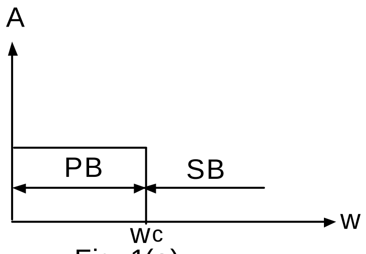


Fig. 1(a)

2. **High pass filter:** A high pass filter is a compolement of a low pass filter in that the frequency range form o to ω_c is the SB and from ω_c to infinity is the PB.

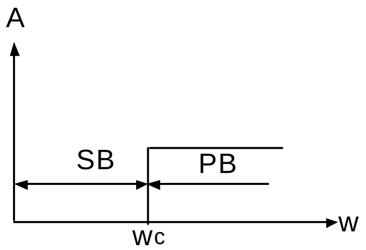


Fig. 1(b)

3. **Band pass filter (BPF):** A BPF is one in which the frequency extending form ω_L (or ω_1) to ω_u (ω_2) are passed while signals at all other frequencies are stopped.

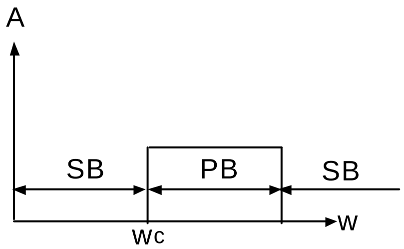


Fig. 1(c)

4. Band stop filter(BSF): A BSF is complement of BPF where signal components at frequencies form ω1 to ω2 are stopped and all others are passed. These filters are sometimes known as "Notch filters".

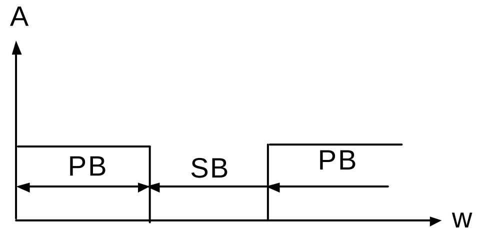
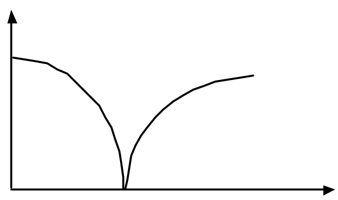


Fig. 1(d)


Notch filter

5. **All pass filters (APF):** It is a filter which passes all range of frequencies , i.e , PB

ranges from o to infinity.

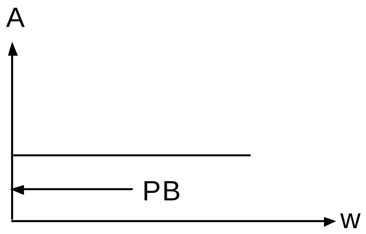
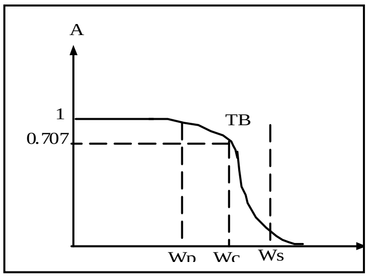


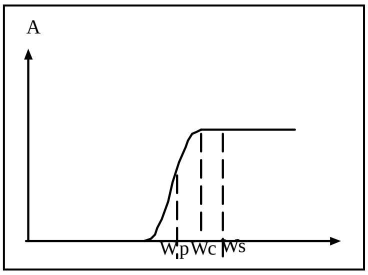
Fig. 1(e)

Non- ideal Characteristics:

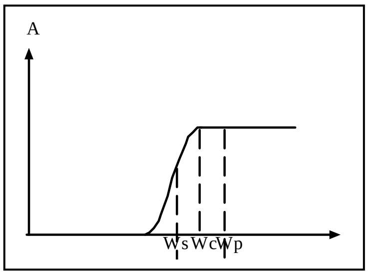
Filter Gain curve Attenuation curve

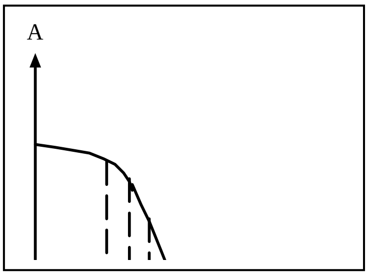
1. LPF





2.HPF





1. From the attenuation curve it to be noted that in the pass band the attenuation is always less then a maximum value. Designated as α_{max}
2. In the stop band the attenuation is always larger then a minimum value designated as α_{min} .
3. Band between PB and SB so defined are known as transition bands. (TB).

Bilinear Transfer function and its poles and zeroes:

We know,

T(s) = P(s)/Q(s) = N(s)/D(s)

$$T(s) = \frac{a_m s^m + a_{m-1}s^{m-1} + ... + a_1 s + a_0}{b_n s^n + b_{n-1}s^{n-1} + ... + b_1 s + b_0}$$

When , m = n = 1, then the T(s) of equation (i) will be bilinear , i.e

$$T(s) = \frac{P(s)}{Q(s)} = \frac{a_1 s + a_0}{b_1 s + b_0}$$

$$= \frac{a_1(s + a_o / a_1)}{b_1(s + b_0 / b_1)}$$

$$= \frac{G(s - z_1)}{(s - p_1)}\left[or\ \ T(s) = \frac{G(s + z_1)}{(s + z_2)} \right]$$

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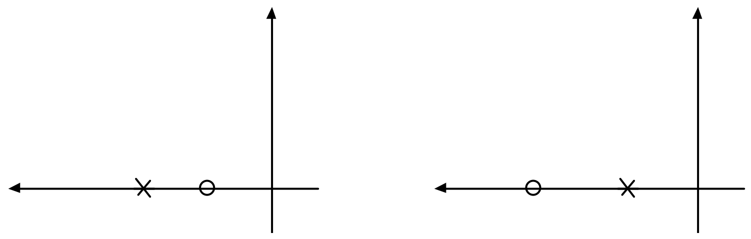
If $z_1 < p_1$ If $p_1 < z_1$



Here, G = a1/b1 = Gain

Z = $-a_0/a_1$ = a zero

P1 = $-b_0/b_1$ = a pole

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Realisation of filter with passive elements:

Let us now see how the bilinear transfer function and its various special cases can be realized with passive elements.

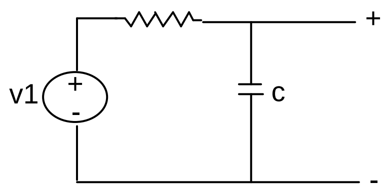


Fig 1.

Plot the magnitude and phase response of the ckt shown in fig (1) and identify the filter.

Solution:

Applying kirchoff's law for fig 1

$$V_1 = R_1 + \frac{1}{L}\int idt................(i)$$

$$V_2 = \frac{1}{L}\int idt......................(ii)$$

Taking laplace transform of equation (i) and (ii)

$$V_1(s) = RI(s) + \frac{1}{cs}I(s)..................(iii)$$

$$V_2(s) = \frac{V_2(s)}{V_1(s)} = \frac{\frac{1}{cs}I(s)}{I(s)\left[R + \frac{1}{cs}\right]}$$

$$= \frac{\frac{1}{cs}}{\frac{Rcs+1}{cs}} = \frac{1}{RC(s+1/RC)}$$

$$= \frac{\frac{1}{RC}}{S+1/RC}$$

$$T(s) = \frac{W_0}{S+W_0}$$

Where, $W_0 = 1/RC$

Now , for magnitude plot,

$T(s) = T(jw) = W_0/(jw+W_0)$

$$\therefore \; |T(jw)| = \frac{w_0}{\sqrt{w^2 + w_o{}^2}}$$

Now when

$W = 0 \quad |T(jw)| = 1$

$W = w_o \quad |T(jw)| = 0.707$

$W = \infty \, , \; |T(jw)| = 0$

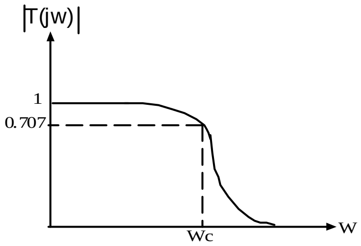


Fig. 2. Magnitude plot

For phase plot:

$\theta(jw) = \tan^{-1}(o/w_0) - \tan^{-1}(w/w_o)$

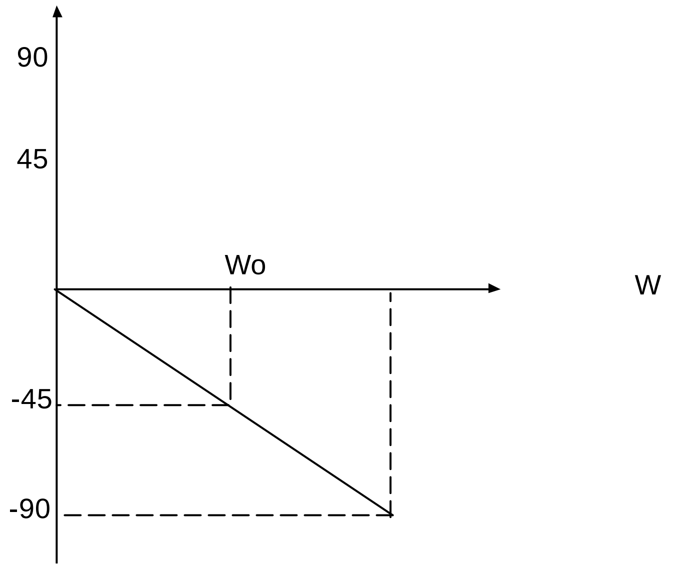
$\theta(jw) = \tan^{-1}(w/w_0)$

When,

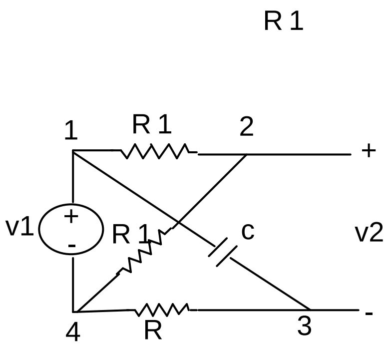
$W = 0 \, , \; \theta(j0) = 0$

$W = w_o \, , \; \theta(jw_o) = -45^\circ$

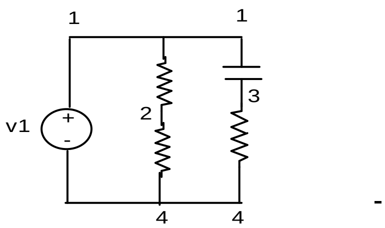
$W = \infty \, , \; \theta(j\infty) = -90^\circ$



2.



Above figure can be modified as:



From figure the potential of node 2, is $V_1/2$ and the potential at node 3 is V_s R/(1+1/cs)

$\therefore V_2 = V_1/2 - V_s R/(1+1/cs)$

V_1/V_2 = ½ - RCS/RCS+1

T(s) = R(S+1- 2RCS)/2(RCS+1) = -{(RCS+1)/2(RCS+1)}

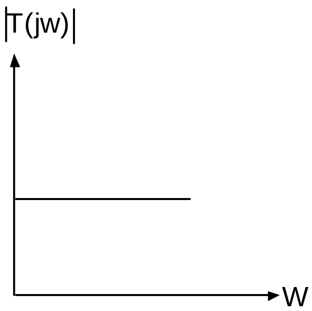
= RC(S+1/RC)/2RC(s+1/RC)

Where W_o = 1/RC

T(jw) = -1/2 {(jw-w_o)/(jw+w_o)}

For magnitude plot ,

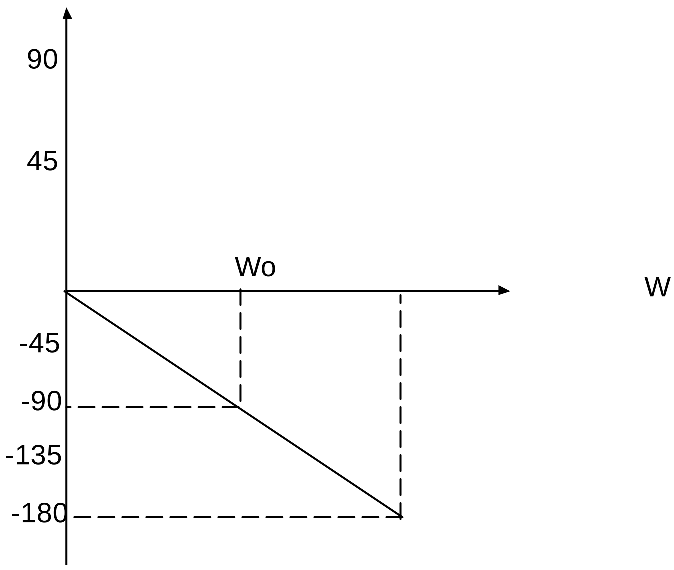
$$|T(jw)| = \frac{1}{2}\frac{\sqrt{w^2 + (w_0)^2}}{\sqrt{w^2 + {w_0}^2}}$$



$|T(jw)| = \frac{1}{2}$

Phase plot:

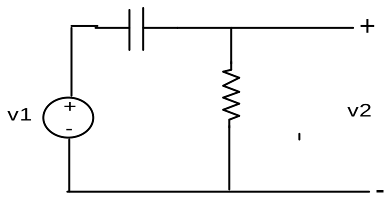
θ (jw) = $\tan^{-1}(-w/w_o)$ - $\tan^{-1}(w/w_o)$
θ (jw) = $-2\tan^{-1}(w/w_o)$
when,
w = 0, θ (jw) = 0
w = 0, θ (jw) = -90 °
w= ∞ , θ (jw) = -180 °



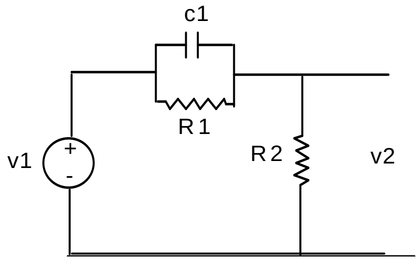
From the magnitude plot, we see that the networking is all pass filter.

Assignment:

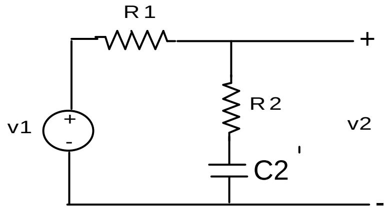
3.



4.



5.



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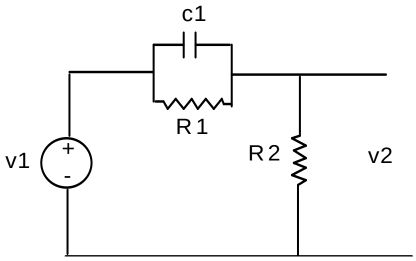
Example :04



From fig (i)

$Y_1 = c_1s+1/R_1 = \dfrac{R_1C_1S+1}{R_1}$

$Z_1 = 1/Y_1 = \dfrac{R_1}{R_1C_1S+1}$

Now applying kirchoff's voltage law, for fig (i).

$V_1 = z_1i+R_2i$

$V_1(s) = (z_1s+R_2)I_1(s)$

And ,

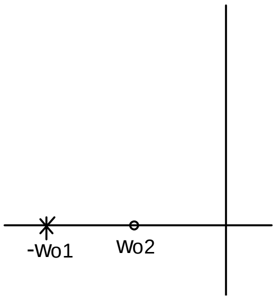
$V_2(s) = R_2I(s)$

$$T(s) = \frac{V_2(s)}{V_1(s)} = \frac{R_2}{Z_1(s)+R_2} = \frac{R_2}{\dfrac{R_1}{R_1C_1S+1}+R_2}$$

$$= \frac{R_2(R_1C_1S+1)}{R_1+R_2R_1C_1S+R_2} = \frac{R_1R_2C_1(S+\dfrac{1}{R_1C_1})}{R_1R_2C_1\left[S+\dfrac{R_1+R_2}{R_2R_1C_1}\right]}$$

$$= \frac{S + \frac{1}{R_1C_1}}{S + \frac{1}{R_2C_1} + \frac{1}{R_1C_1}}$$

Or, T(s) = $\frac{S+\omega_{01}}{S+\omega_{02}} = \frac{S-(-\omega_{01})}{S-(-\omega_{02})}$



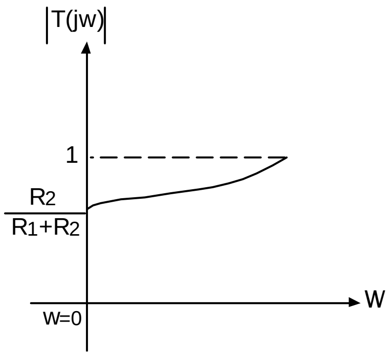
And , $\omega_{02} > \omega_{01}$
$or, \ -\omega_{02} < \omega_{01}$

For Magnitude plot:

T(jw) = $\frac{jw+w_{01}}{jw+w_{02}} = \frac{\sqrt{w^2 + w_{01}{}^2}}{\sqrt{w^2 + w_{02}{}^2}}$

Now at w= 0, $|T(j0)| = \frac{w_{01}}{w_{02}} = \frac{R_2}{R_1+R_2}$

At w = ∞, $|T(j\infty)| = \frac{w_{01}}{w_{02}} = 1$



For Phase plot,

T(jw) = $\frac{jw+w_{01}}{jw+w_{02}}$

Where, w_{01}= $1/R_1C_1$

$W_{02} = 1/R_1C_1 + 1/R_2C_2$

Therefore, θ(jw) = $\tan^{-1}\left(\frac{w}{w_{01}}\right) - \tan^{-1}\left(\frac{w}{w_{02}}\right)$

$\theta(jw) = \theta_z - \theta_p$

Since direct phase plot of above expression is very complicated, we will go it by indirect method. First we will plot the zero phase and then the pole phase and finally find the net pole – zero phase.

Zero plot (θ_z)

$$\theta(z) = \tan^{-1}\left(\frac{w}{w_{01}}\right) = \tan^{-1}(wR_1C_1)$$

Now at w = 0

$\theta(z) = \theta(j0) = 0$

$\theta(z) = \theta(jw_0) = 45^\circ$

Now at $w = \infty$

$\theta(j\infty) = 90^\circ$

Pole plot (θ_p)

$\theta(p) = \tan^{-1}(w/w_{01})$

$$= \tan^{-1}\left(\frac{w}{\frac{1}{R_1C_1} + \frac{1}{R_2C_1}}\right)$$

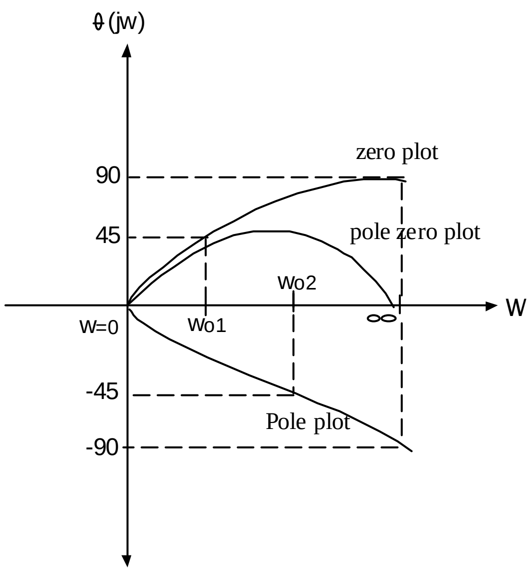
Now at, w = 0

$\theta_p = \theta(j0) = 0$

at $w = w_{02}$

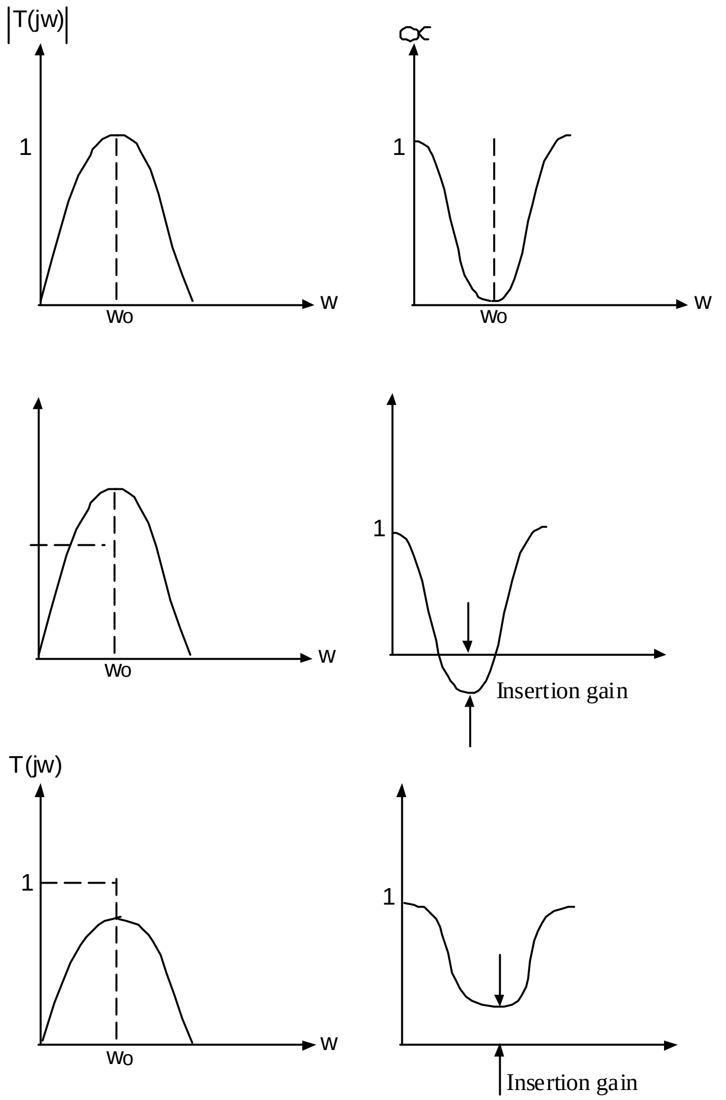
$\theta_p = \theta(w_{02}) = 45^\circ$

at $w = \infty$, $\theta_p = \theta(j\infty) = 90^\circ$



Thus the magnitude response of the above network shown that it is a high pass filter with dc gain $R_2/(R_1+R_2)$ and phase plot signifies it is leading type.

Insertion Gain and insertion loss:



One of the important factor that should be consider in design is that the minimum value of α should be zero degree. But this is not true in practical case since we are using active element , this need not be the case because the active element may provided the gain greater than one (1). If it is necessary to meet the specification exactly then it will be necessary to provide ck t to reduce the gain. We call this unwanted gain as the insertion gain. On the other hand there is a loss in the components of passive filter so it provides access attenuation and we call this loss as insertion loss. To overcome this problem additional compensation circuit is required.

Chapter- 2

Normalization and Renormalization:

In most of the cases we consider the values of R, L S& C to be the order of unity. It is very difficult to built the capacitor of 1 f and inductor of 1 H . Besides this the practical values of capacitors available in the electronic circuit is of the order of microfarad or Pico farad. The circuit considered so for have normalized elemental values but practically these values are not realizable. So we perform scaling to get the realizable components.

There are mainly two reasons for resorting the normalized design.

1. Numerical computation become simple and it is easier to manipulate the numbers of the order of unity.
2. If we have the normalized design of the filter then it is easy to generate the

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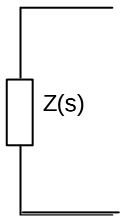
filter of similar characteristics of varying center frequency and impedance level without redesigning the whole circuit.

The actual or the required elemental values of the Filter ckt which is obtained after scaling is called demoralized values of the circuit.

Scaling: While designing the ckt sometimes the value of components may not be available so we change them with the available one, which is called scaling. To obtained the elemental values of the required filter we amplitude and frequency scale the normalized design.

Types of scaling:

1. **Impedance (Magnitude or amplitude) scaling:** In this scaling, the magnitude of the impedance is increased or decreased. To scale in magnitude , z(s) (the impedance) is multiplied by a constant factor K_m .



If K_m . 1, then it is called scale up.
If $K_m < 1$, then it is called scale down.

Let, R_{old} = old value of Resistor.
L_{old} = old value of inductor
C_{old} = old value of capacitor.

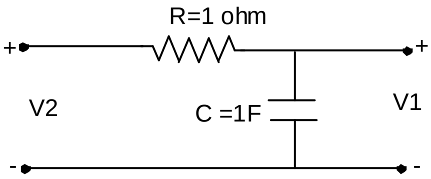
The new values of R, L and C are given by

$R_{new} = K_m R_{old}$ …………(i)
Also,
$X_L K_m = L_{old} S K_m = (K_m L_{old})S = L_{new} S$
$L_{new} = K_m L_{old}$ ……………….(ii)
Again,

$$X_c K_m = 1/c_{old}\, s \,.\, K_m = \frac{1}{\left(\frac{C_{old}}{K_m}\right)s} = \frac{1}{C_{new}.S}$$

$$C_{new} = \frac{C_{0ld}}{K_m} \quad \text{………………………(iii)}$$

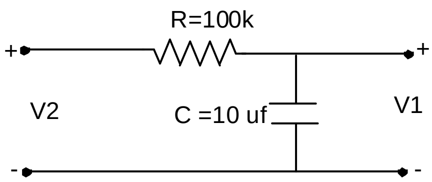
Example 01:
Perform Impedance scaling to the following network.



Solution:

R_{old} = 1 Ω
C_{old} = 1 F
Now , let us assume that,
C_{new} = 10 μF

Note: Generally we assume new value of capacitor 1μF or 10 μF.
We know that
$C_{new} = C_{old}/K_m$
$K_m = C_{old}/C_{new}$ = 1F/10 μF = 10^5
Therefore, $R_{new} = K_m .R_{old}$
= 10^5 * 1 Ω
R_{new} = 100K



Fig(ii) scaled ckt.

The transfer function for fig. (i) ,
$T_{old}(s) = 1/(s+1)$

And, $T_{new} = \dfrac{\dfrac{1}{R_{new}C_{new}}}{S + \dfrac{1}{R_{new}C_{new}}}$ = 1/s+1

Thus we see that there is no change in the following transfer function while doing magnitude scaling.

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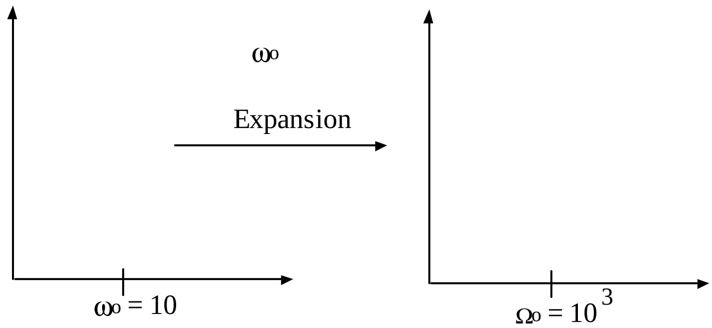
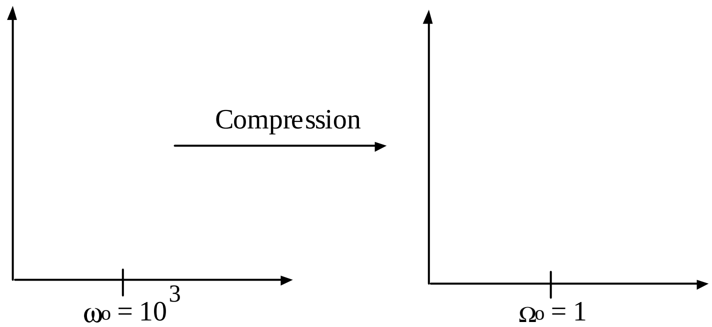
2. Frequency scaling:
In frequency scaling our objective is to scale the frequency without affecting the magnitude of the impedance , i.e
$Z_L = (= X_L) = LS = jWL$
$|Z_L| = WL$ is a constant.
Similarly,
$Z_c (= X_c) = 1/cs = 1/ jwc$

$|Z_c| = \frac{1}{wc}$ is constant.

To do so any change in w must be compensated by corresponding change in L and c
If, w = old corner frequency
Ω = new corner frequency.

$\Omega = K_f w$
Where,
K_f = frequency scaling factor.
If $K_f > 1$, then it is called expansion scaling
If, $K_f < 1$, then it is called compression scaling.





Also, if T(jw) is old Transfer function, then the new transfer fucnti is $T(j\Omega)$ $= T(jK_f w)$

The resistance is unaffected by frequency scaling , i.e
$R_{new} = R_{old}$ …………….(v)

For inductor,
$X_l = Ls = jwL = jwk_f \cdot L/k_f$

Or, $X_L = j(wk_f)\,(L_{old}/k_f)$ since, $L = L_{old}$

$= j\Omega\,(L_{old}/k_f)$

$L_{old} = L_{old} / K_f$ ………….(vi)

For capacitor,
$C_{new} = C_{old}/ k_f$............................(vii)

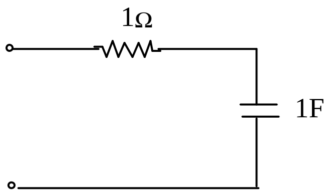
3. Both magnitude and Frequency scaling:

It is not necessary that we scale magnitude and scale in frequency separately. We can do both at once. Cobining all the above equations.

$R_{new} = K_m R_{old}$ …………….(Viii)
$L_{new} = K_m/k_f .\ L_{old}$ ……………(ix)
$C_{new} = C_{old}/K_m.k_f$ …………(x)

These three equations are know as element scaling equations.

Example 01:



Solution:
$W_0 = 1$, $\Omega = 1000$
Therefore, $k_f = \Omega_o/w_o = 1000$
Now we know that
$C_{new} = C_{old}/k_f = 1F/1000 = 1$ mF
And , $R_{new} = R_{old} = 1\ \Omega$

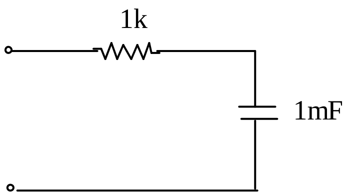


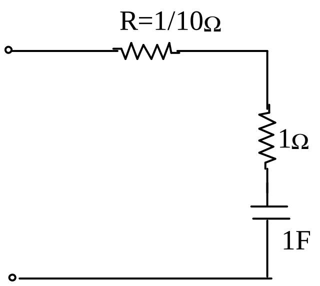
Fig (ii): after frequency scaling.

Now,

$$T_{old}(S) = \frac{\frac{1}{R_{0ld}C_{old}}}{s + \frac{1}{R_{old}C_{old}}} = \frac{1}{s+1}$$

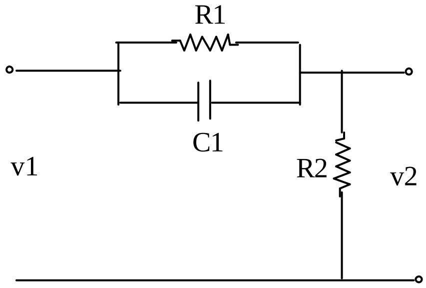
And, $$T_{new}(s) = \frac{\frac{1}{R_{new}C_{new}}}{s + \frac{1}{R_{new}C_{new}}} = \frac{10}{s+10}$$

Example 02:



Perform frequency scaling with $\Omega_o = 1\ \Omega$

Example 03:



T(s) = (s+0.5)/(s+3)

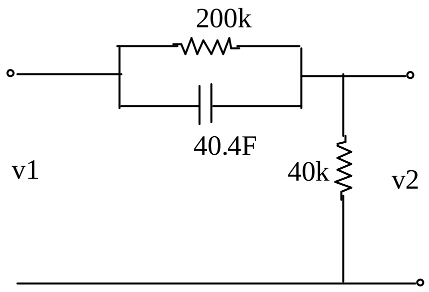
Perform magnitude and frequency scaling separately with $w_o = 3$ and $\Omega_0 = 300$.

Solution:
The transfer function of the above figure is

$$T(s) = \frac{s + \frac{1}{R_1 C_1}}{s + \frac{1}{R_1 C_1} + \frac{1}{R2 C_1}} \quad \ldots\ldots\ldots\ldots\ldots.(i)$$

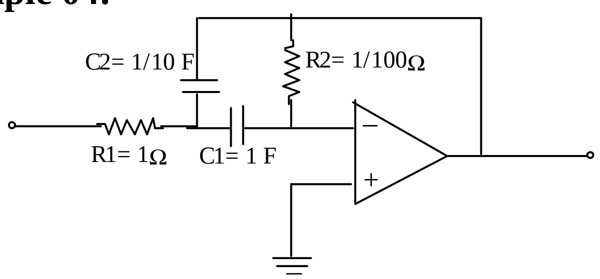
But given ,
T(s) = (s+0.5)/(s+3) ……………….(ii)
Comparing equation (i) and (ii)

$1/R_1C_1 = 0.5$
$R_1C_1 = 2$ ……………..(iii)
Again, $(1/R_1 + 1/R_2)1/C_1 = 3$…………..(iv)
Let , $C_1 = 1$ F
For equation (iii) $R_1 \times 1 = 2$
$R_1 = 2\ \Omega$
Therefore from equation (iv)
$(1/2 + 1/R_2)\ 1/2 = 3$
Therefore, $R_2 = 2/5\ \Omega$
In order to perform magnitude scaling
$R_{1old} = 2\ \Omega$
$R_{2old} = 2/5\ \Omega = 0.4\ \Omega$
$C_{old} = 1$ F
Say, $C_{1new} = 10\ \mu F$
Then, $K_m = C_{old}/C_{new} = 1F/10\ \mu F$
$K_m = 10^5$
Therefore, $R_{new} = k_m R_{2old} = 10^5 \times 0.4\ \Omega = 40$ k
The selected ckt will be :



Again for frequency scaling,
$W_o = 3$, $\Omega_0 = 3000$
Therefore , $k_f = \Omega_o / w_o = 3000/ 3 = 1000$
Therefore, $R_{1new} = R_{1old} = 2\ \Omega$
$R_{2new} = R_{2old} = 0.4\ \Omega$
$C_{1old} = C_{1old}/k_f = 1F/ 1000 = 1$ mF.

Example 04:



Perform magnitude scaling to the ckt given.
Note: Take 'C_{new}' as the new value of capacitor for 'C_{old}' where 'C_{old}' represents the largest value in the circuit.

Solution:
Here, $R_{1old} = 1\ \Omega$
$R_{2old} = 2\ \Omega$
$C_{1old} = 1$ F
$C_{2old} = 1/10$ F.
Take, $C_{new} = 10\ \mu F$.
Then for, magnitude scaling,
$C_{new} = C_{old}/k_m$
$K_m = C_{1old}/ C_{1new} = 1F/ 10\ \mu F = 10^5$
Therefore, $C_{2new} = C_{2old}/k_m = 0.1\ F/ 10^5$
$C_{2new} = 1\ \mu F$
Similarly,
$R_{1old} = k_m.\ R_{1old} = 10^5 \times 1\ \Omega = 100$ k
$R_{2new} = k_m.\ R_{2old} = (1/100).\ 10^5\ \Omega = 1$ k.

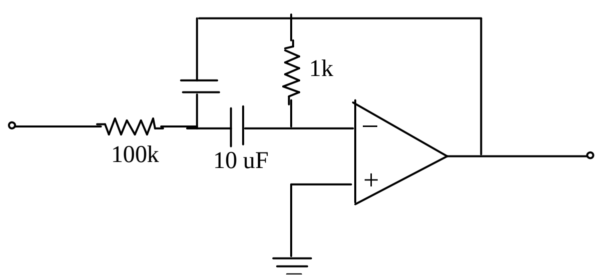


Fig: Magnitude Scaling Ckt.

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Chapter: 3

One port and two port passive network:

Positive real function: The filter circuit is complex transfer function that may be realizable depending upon weather the transfer function exhibits PRF properties. I the transfer function is PRF only ckt is realizable. There are two types of passive network : [i] one port network [ii] Two port network.

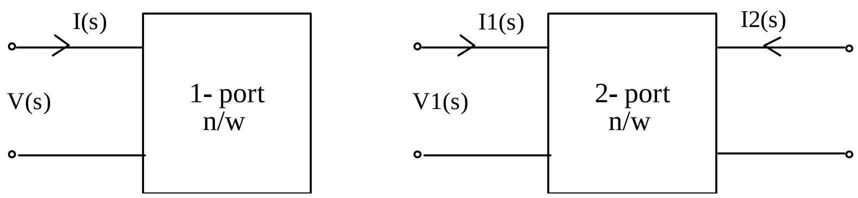
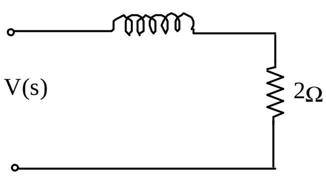


Fig. 1(a) one port n/w Fig. 2(b) two port n/w

One port network: Let us suppose of fig of 1(a),
Then, z(s) = V(s) / I(s)
If V(s) = 3s+2
I(s) = 1
Then, z(s) = 3s+2
= Ls +R



Thus , the function is realization but if, z(s) = 3s-2 , then it is not realizable.

Date: 2065/ 5/10
Why? (छुटेको छ)

(i) If F(s) denote the function in S-domain, the F(s) indicates either driving point impedance or driving admittance. Which ever is concern to us.
(ii) F(s) should be for real value of S.
(iii) The value of F(s) must be greater than or equal to zero. i.e $Re[f(s)] \geq 0$.

Thus in brief a PRF must be real and +ve .
If F(s) = LS = jWL ⟹ L must be +ve.
F(s) = 1/CS = 1/jwc ⟹ C must be +ve
F(s) = R ⟹ R must be +ve.

Properties of Passive n/w.
A passive network is one
(i) The element of which one are +ve and real.

(ii) The average Power dissipated (APD) by the n/w. for a sinusoidal i/p must be +ve. For one port n/w APD = 1/2 Re[z(s)][I(s)]$^2 \geq 0$

Properties of PRF:

1. If F(s) is +ve and real , then 1/F(s) is also +ve and real.

2. The sum of DRFS is always PRF but the difference may not be PRF.

Example: $Z_1(s)$ = 5s+ 3 (PRF)

$Z_2(s)$ = 2s+ 5 (PRF)

Then, $z_1(s)+z_2(s)$ = 7s+8 (PRF)

But, $Z_1(s) – Z_2(s)$ = 3s-2 (not PRF)

3. The Poles and zero's of PRF cannot be in the right half of the S-Plain.

4. Only poles with real residues can exists on the jw axis.

Example: F(s) = 6s/($s^2+\sigma^2$)

In this case, S = $\pm\sigma\ j$

Residue = σ real and +ve.

5. The poles and zeroes of PRF Occurs in pairs.

6. The highest power of numerator and denominator polynomial may differ atmost by unity.

Example: $\dfrac{S^5+4S^4+3S^3+3S^2+3S^1+2}{S^6+4S^4+2S^3+3S^2+3K}$

7. The lowest power of numerator and denominator polynomial may differ atmost by unity.

Example: $\dfrac{S^5+4S^4+3S^3+3S^2+3S}{S^6+4S^4+2S^3+3S^2+3K}$

8. The real part of F(s) must be greater than or equal to zero. i.e Re[F(s)] ≥ 0

But , if Re[F(s)] = 0 , then the ckt do not consist resistive components. Hence only capacitive and inductive components are presents. Hence only capacitive and inductive components are present . Such a n/w whose transfer function satisfies this condition is known as lossless n/w.

Example: Determine weather the function is PRF.

(i) z(s) = $2s^2+5/s(s^2+1)$

Hence , z(s) = $2s^2+5/s(s+1)$

A/s + Bs/(s^2+1) = A/s + B/ (s^2+1)/s

$$A = \frac{2s^2+5}{s(s+1)}.s\Big|_{s=0}$$

$$B = \frac{2s^2+5}{s(s^2+1)}.\frac{(s^2+1)}{s}\Big|_{s^2}$$

$$= \frac{2(-1)+5}{(-1)} = -3$$

Z(s) = 5/3 + -3s/(s^2+1)

Here, (-3) , the residues (s^2 = -1) is –ve , therefore z(s) is not PRF.

(ii) z(s) = $\dfrac{(s+1)(s+4)}{(s+1)(s+3)} = \dfrac{s(s+4)+2(s+4)}{s(s+3)+1(s+3)}$

$$= \frac{s^2 + 6s + 8}{s^2 + 4s + 3}$$

$$= 1 + \frac{2s + 5}{(s+1)(s+3)}$$

$$= z_1(s) + z_2(s)$$

Where $z_2(s) = \frac{2s+5}{(s+1)(s+3)} = \frac{A}{s+1} + \frac{B}{S+3} = \frac{3/2}{s+1} + \frac{1/2}{s+3}$

Therefore, $z(s) = 1 + \frac{3/2}{s+1} + \frac{1/2}{s+3}$

It is not PRF.

(iii) $z(s) = \frac{8s^3 + 4s^2 + 3s + 1}{8s^3 + 3s}$

(iv) $Y(s) = \frac{s^2 + 2s + 8}{s(s+4)}$

Basic ckt Synthesis Techniques:

Any one port n/w each can be represented by either admittance function Y(s) or impedance function z(s) . i.e

$$F(s) = \frac{a_n s^n + a_{n-1}s^{n-1} + a_{n-2}s^{n-2} + \dots\dots\dots\dots + a_1 s + a_0}{b_m s^m + b_{m-1}s^{m-1} + b_{m-2}s^{m-2} + \dots\dots\dots\dots + b_1 s + b_0}$$

$$= \frac{P(s)}{Q(s)}$$

$$= \frac{N(s)}{D(s)}$$

$$= \frac{Z(s)}{P(s)}$$

Design of LC Ckt . (Loss less ckt):

Consider a impedance function as

$$Z(s) = \frac{E_n(s) + O_n(s)}{E_m(s) + O_m(s)}$$

Where $E_n(s)$ and $O_m(s)$ denote the even parts of numerator and denominator respectively and $O_n(s)$ and $O_n(s)$ denote odd part.

$$Z(s) = \frac{s^5 + s^4 + s^3 + s + 1}{s^6 + s^5 + s^4 + s^3 + s^2 + s + 1} = \frac{N(s)}{Q(s)}$$

$$= \frac{\underbrace{(s^4 + s^2 + 1)}_{E_n(s)} + \underbrace{(s^5 + s^3 + 5)}_{O_n(s)}}{\underbrace{(s^6 + s^4 + s^2 + 1)}_{E_m(s)} + \underbrace{(s^5 + s^3 + 1)}_{O_m(s)}}$$

For the loss less function , it is to be noted that,

Re[z(s)] = 0 ………….(i)

Now, $z(s) = \frac{E_n(s) + O_n(s)}{E_m(s) + O_m(s)} \times \frac{E_m(s) - O_m(s)}{E_m(s) - O_m(s)}$

$$= \frac{E_n(s)E_m(s) + O_n(s)E_m(s) - E_n(s)O_m(s) - O_n(s)O_m(s)}{E_m{}^2(s) - O_m{}^2(s)}$$

$$= \frac{E_n(s)E_m(s) + O_n(s).O_m(s)}{E_m{}^2(s) + O_m{}^2(s)} + \frac{O_n(s)E_m(s) - E_n(s)O_m(s)}{E_m{}^2(s) - O_m{}^2(s)}$$

$$= \text{Re}[\ z(s)] = \frac{E_n(s)E_m(s) - O_n(s).O_m(s)}{E_m{}^2(s) - O_m{}^2(s)} \ \ldots\ldots\ldots\ldots(ii)$$

Therefore from equation (i) and (ii).

$$\frac{E_n(s)E_m(s) - O_n(s).O_m(s)}{E_m{}^2(s) - O_m{}^2(s)} = 0$$

$$E_n(s)E_m(s) - O_n(s).O_m(s) = 0$$

$$E_n(s)E_m(s) = O_n(s).O_m(s)$$

$$\frac{E_m(s)}{O_m(s)} = \frac{O_n(s)}{E_m(s)} \ \ldots\ldots\ldots\ldots\ldots(iii)$$

The above equation (iii) indicates that LC ckt is even to odd (or odd) to even function.

Properties of LC Ckt:

1. $F(s) = \dfrac{a_n s^n + a_{n-2}s^{n-2} + a_{n-2}s^{n-4} + \ldots\ldots\ldots\ldots + a_0}{b_m s^m + b_{m-2}s^{m-2} + b_{m-4}s^{m-4} + \ldots\ldots\ldots\ldots + b_0}$

The coefficients a_n and b_m must be real and +ve and F(s) must be even to odd or odd to even function.

2. The highest power of numerator and denominator can differ atmost by unity (in this case it is 2). So does the lowest power.
3. The succeeding power of 's ' in numerator and denominator must differ by the order of 2 all the way through . Example: $\dfrac{s^4 + 17s^2 + 165s^0}{s^3 + 4s}$
4. The poles and zeros must be alternatively placed on the jw axis and lie only on the imaginary axis.
5. There must be either a pole or a zero at the origin.

Example: Test whether the following function is LC.

(i) z(s) = K $(s^2+1)(s^2+5)/(s^2+2)(s^2+10)$ k>0

It is not LC ckt function because,

1. There is neither pole or zero at the origin though the pole zero are alternatively placed on the imaginary axis.
2. It is not even to odd or odd to even function.

(ii) Z(s) = $z(s^2+1)(s^2+9)/s(s^2+4)$

(iii) Z(s) = k $s(s^2+4)/(s^2+1)(s^2+3)$, k> 0

(iv) Z(s) = $s^5+4s^3+5/(4s^4+s^2)$

Date: 2065/5/12

Design of LC ckt by Foster's Method:

In this case ,

$$F(s) = \frac{k_0}{S} + \frac{2k_i s}{s^2 + w_i{}^2} + \ldots\ldots\ldots\ldots + k_\infty s \ \ldots\ldots\ldots\ldots\ldots\ldots\ldots(i)$$

This equation may represent z(s) or Y(s)

Case I : (i.e when F(s) = z(s))

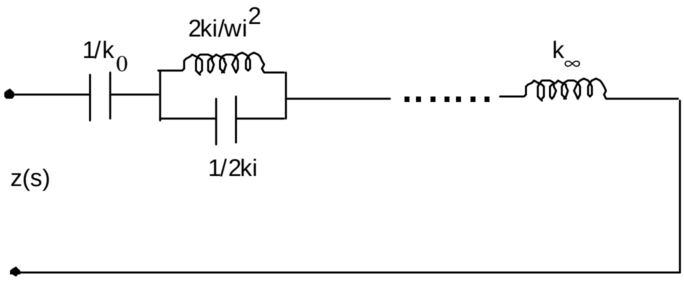
Then,

$$Z(s) = \frac{k_0}{S} + \frac{2k_i s}{s^2 + w_i^2} + ... + k_\infty s$$

Here,

- k_o/s will represent a capacitive reactance of $1/k_o$ F.
- $2k_i(s)/(s^2+w^2)$ will represent LC parallel combination.

Having capacitor of value $1/2k_i$ F and inductor of value $2k_i/w_i^2$. Thus the final circuit will be:



This method of circuit synthesis is known as foster impedance or series or 1st method for LC ckt.

Case – II

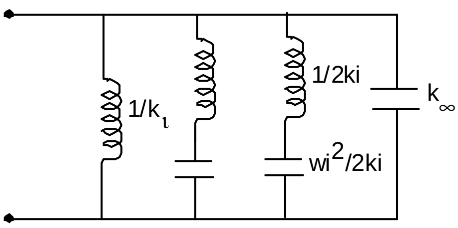
In this case , F(s) = Y(s) , then equation (i) becomes

$$Y(s) = \frac{k_0}{S} + \frac{2k_i s}{s^2 + w_i^2} + ... + k_\infty s$$

Here,

- K_O/s represents admittance of inductor having value of $1/k_o$ H.
- $K_\infty s$ represent admittance of capacitor having value K_∞ F.
- $2k_i(s)/s^2+w^2$ represents admittance of series LC combination having inductor of value $1/2k_i$ H and capacitor value $w_i^2/2k_i$

The ckt can be realize as :



This method of circuit synthesis is known as foster admittance or parallel or 2nd method for LC ckt.

Example 01: Design a Foster series n/w for the following n/w.

$$F(s) = \frac{s(s^2 + 4)}{2(s^2 + 1)(s^2 + 9)}$$

Solution:

It is Foster's series n/w

$$F(s) = z(s) = \frac{s(s^2+4)}{2(s^2+1)(s^2+9)}$$

Now, $z(s) = \dfrac{s(s^2+4)}{2(s^2+1)(s^2+9)} = \dfrac{As}{s^2+1} + \dfrac{Bs}{s^2+9}$

Where, A $= \left.\dfrac{s(s^2+4)}{2(s^2+1)(s^2+9)} \cdot \dfrac{(s^2+9)}{s}\right|_{s^2=-1}$

$= \dfrac{-1+4}{2(-1+9)} = 3/16$

Therefore, A = 3/16

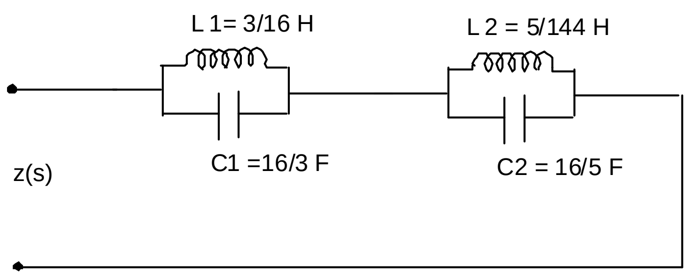
And B = $\left.\dfrac{s(s^2+4)}{2(s^2+1)(s^2+9)} \cdot \dfrac{(s^2+9)}{s}\right|_{s^2=-9}$

$= \dfrac{-9+4}{2(-9+1)} = \dfrac{-5}{2\times -8} = \dfrac{5}{16}$

Therefore, B = 5/16

$$z(s) = \frac{(3/16)s}{s^2+1} + \frac{(5/6)s}{s^2+9} = z_1(s) + z_2(s)$$

The ckt will be as follows.



- The first part of z(s) (i.e $z_1(s)$) represents parallel LC combination having inductor L_1 of value 3/16 H and capacitor of value 16/3 F.
- The 2nd part of z(s) (i.e $z_2(s)$) represents parallel LC combination having inductor L_2 of value 5/144 H and capacitor C_2 of value 16/5 F.

Example 02: Design Foster parallel n/w for the function

$$F(s) = \frac{s(s^2+4)}{2(s^2+1)(s^2+9)}$$

Solution:

It is Foster's parallel n/w

$$F(s) = Y(s) = \frac{s(s^2+4)}{2(s^2+1)(s^2+9)}$$

Now, $z(s) = \dfrac{s(s^2+4)}{2(s^2+1)(s^2+9)} = \dfrac{As}{s^2+1} + \dfrac{Bs}{s^2+9}$

Where, A $= \left.\dfrac{s(s^2+4)}{2(s^2+1)(s^2+9)} \cdot \dfrac{(s^2+9)}{s}\right|_{s^2=-1}$

$$= \frac{-1+4}{2(-1+9)} = 3/16$$

Therefore, A = 3/16

And $B = \frac{s(s^2+4)}{2(s^2+1)(s^2+9)} \cdot \frac{(s^2+9)}{s} \Big|_{s^2=-9}$

$$= \frac{-9+4}{2(-9+1)} = \frac{-5}{2\times -8} = \frac{5}{16}$$

Therefore, B = 5/16

$$Y(s) = \frac{(3/16)s}{s^2+1} + \frac{(5/6)s}{s^2+9} = Y_1(s) + Y_2(s)$$

The ckt will be as follows:

Figure:

- The first part of Y(s) (i.e $Y_1(s)$) represents series LC combination having inductor L_1 of value 16/3 H and capacitor of value 16/3 F.
- The 2nd part of Y(s) (i.e $Y_2(s)$) represents series LC combination having inductor L_2 of value 16/5 H and capacitor C_2 of value 144/5 F.

Example 03: Design Foster parallel n/w for the function $F(s) = \frac{2(s^2+1)(s^2+9)}{s(s^2+4)}$

Solution:

It is Foster Parallel ,

$$F(s) = Y(s) = \frac{2(s^2+1)(s^2+9)}{s(s^2+4)}$$

$$= \frac{2s^4+20s^2+18}{s^3+4s}$$

$S^3+4s\ )2s^4+20s^2+18(\ 2s$

$\quad 2s^4+8s^2$

$\quad\quad 12s^2+18$

Therefore, $Y(s) = 2s + \frac{12s^2+18}{s^3+4s}$

$$= 2s + \frac{12s^2+18}{s(s^2+4)}$$

$Y(s) = Y_1(s) + Y_2(s)$

Now $Y_2(s) = 2s + \frac{12s^2+18}{s(s^2+4)} = \frac{A}{s} + \frac{Bs}{s^2+4} = \frac{9/2}{s} + \frac{(15/2)s}{s^2+4}$

$Y(s) = 2s + \frac{9/2}{s} + \frac{(15/2)s}{s^2+4} = Y_1(s) + Y_2(s) + Y_3(s)$

Here $Y_1(s) = 2s$, so $C_1 = 2$ F

$$Y_2(s) = \frac{9/2}{s}$$, So, $L_1 = 2/9$ H

And $Y_3(s) = \frac{(15/2).s}{s^2+4}$

$L_2 = 2/15$ H

$C_2 = 8/15$ F

Therefore, The final ckt will be

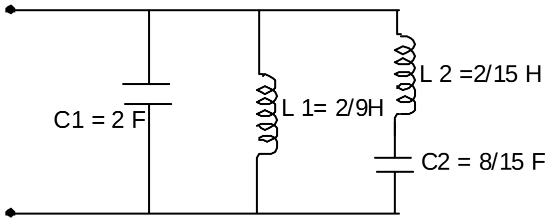


Fig. Foster's parallel n/w of LC ckt.

Assignment:

1. z(s) = $\frac{2(s^2+1)(s^2+9)}{s(s^2+1)}$
2. Y(s) = $\frac{2(s^2+2)(s^2+4)}{(s^2+3)(s^2+1)}$

Date: 2065/5/17

Continued Fraction method or cauer method for LC Ckt

1. case- I

It is removed by successive removal of pole at ∞. The ckt will be as follows:

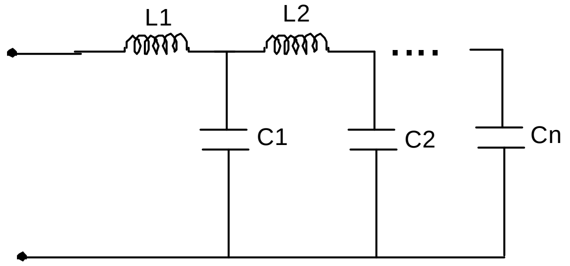


Fig. For F(s) = z(s)

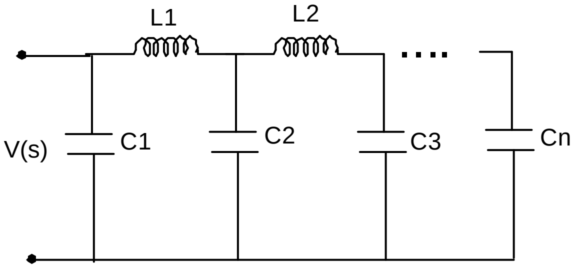


Fig. For F(s) = Y(s)

Example 01: Synthesis the following function in cauer form.

$$Z(s) = \frac{2s^5 + 12s^3 + 16s}{s^4 + 4s^2 + 3}$$

Solution:

In cauer n/w we proceed as follows:

$S^4 + 4s^2 + 3)\ 2s^5 + 12s^3 + 16s\ (2s \qquad z_1(s)$

$\quad 2s^5 + 8s^3 + 6s$

$4s^3 + 10s)\ s^4 + 4s^2 + 3\ (s/4 \qquad Y_2(s)$

$\quad S^4 + 10s^2/4$

$3S^2/2 + 3)\ 4S^3 + 10S\ (8s/3 \qquad Z_3(s)$

$\quad 4S^3 + 8S$

$2S)\ 3S^2/2 + 3\ \ (3s/4 \qquad Y_4(s)$

$\quad 3s^2/2$

$3)\ 2s\ (2s/3 \qquad Z_5(s)$

$\quad 2s$

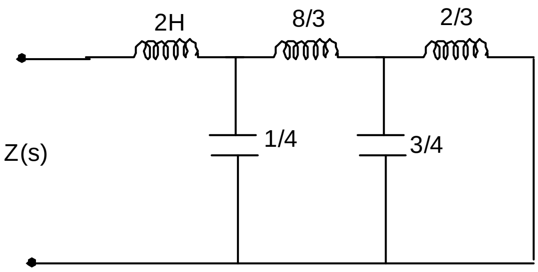


Fig. Cauer n/w for LC series ckt

Example: 02: $Y(s) = \frac{2s^5 + 12s^3 + 16s}{s^4 + 4s^2 + 3}$

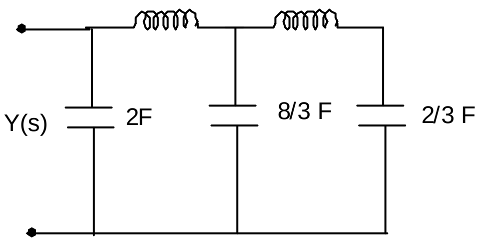


Fig: Cauer n/w for LC parallel ckt.

Example:03: Synthesis the following ckt in cauer form.

(i) $Y(s) = \frac{s(s^2+2)(s^2+4)}{(s^2+1)(s^2+3)}$ (ii) $Z(s) = \frac{s(s^2+2)(s^2+4)}{(s^4+1)(s^2+3)}$

Cauer –II:

This is the case of removal of pole at origin.

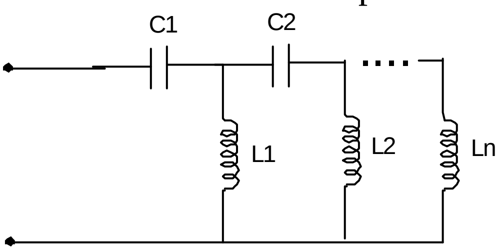


Fig. Caure II n/w for LC series ckt.

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Example:01: Synthesize the following function in cauer form.

$$Z(s) = \frac{s^4 + 4s^2 + 3}{2s^5 + 12s3 + 16s}$$

Solution:

Since Z(s) is the case of pole at origin (i.e s = 0) z(s) can be rewrite as:

$$Z(s) = \frac{3 + 4s^2 + s^4}{16s + 12s^3 + 2s^5}$$

$16s+12s^3+2s^5$) $3+4s^2+ s^4$ ($3/16s$ $z_1(s)$

$3+9s^2/4+ 3s^4/8$

$7s^2/4+5s^4/8$) $16s+12s^3+2s^5$ ($64/7s$ $Y_2(s)$

$16s+40s^3/7$

$44s^3/7+2s^5$) $7s^2/4+5s^4/8$ ($49/176s$ $Z_3(s)$

$7s^2/4+44s^4/88$

$3s^4/44$) $44s^3/7 +2s^5$ ($(44)^2/21s$ $Y_4(s)$

$44s^3/7$

$2s^5$) $3s^2/44$ ($3/88s$ $Z_5(s)$

$3s^2/44$

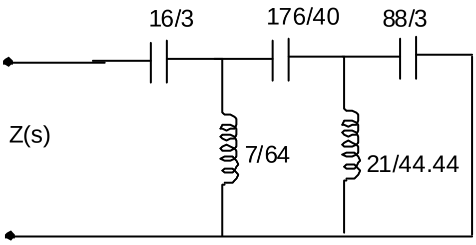


Fig. Cauer II n/w for LC ckt

Example:02: $Y(s) = \frac{s^4 + 4s^2 + 3}{2s^5 + 12s^3 + 16s}$

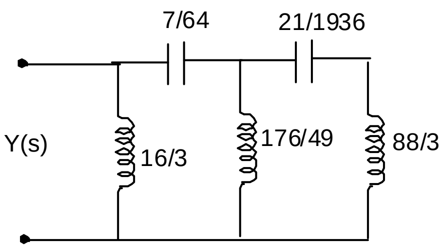


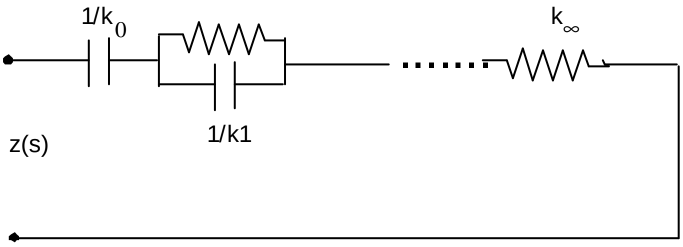
Fig. Caure II n/w for parallel LC ckt.

R-C one port n/w: (R-C impedance /R-L admittance)

1. Foster 1^{st} method:

In this case,

F(s) = z(s) , gives R-C impedance n/w.



Foster method defines F(s) as

$F(s) = z(s) = k_o/s + k_1/(s+\sigma_1) + k_2/(s+\sigma_2) + \ldots\ldots\ldots + k_\infty$

Here,

- k_o/s represent capacitive reactance having capacitor of value $1/k_o$ F.
- k_∞ represent resistor of value $k_\infty\,\Omega$.
- $k_i/(s+\sigma_i)$ represents RC parallel in which the resister has a value of $k_i/\sigma_i\ \Omega$ and a capacitor has value of $1/k_i$ F.

Properties of RC impedance N/w:

1. the poles of RC –impedance n/w are on the –ve real axis.
2. As in LC ckt, residues of poles (k_is) are real and +ve i. z(s) must be PRF.
3. At two critical frequencies i.e when s = o , i.e $\sigma = 0$ when $s = \infty$ i.e $\sigma = \infty$
4. $z(0) = \infty$ if C_0 is present
   $= \sum R_i$, if C_0 is missing
5. $z(\infty) = k_\infty$, R_∞ is present
   $= 0$, R_∞ is missing
6. $z(0) \geq z(\infty)$ is always true.
7. The critical frequency nearest to the origin must be a pole.
8. The poles and zeroes must be alternatively placed.

Example:01 State giving reasons which of the following if not RC impedance.

(a) $Z(s) = \frac{(s+1)(s+4)(s+9)}{s(s+2)(s+5)}$

(b) $Z(s) = \frac{(s+1)(s+8)}{(s+2)(s+4)}$

(c) $Z(s) = \frac{(s+2)(s+4)}{(s+1)}$

(d) $Z(s) = \frac{(s+1)(s+2)}{s(s+3)}$

Example:02: Synthesis the following function in Foster series form: $F(s) = \frac{6(s+2)(s+4)}{s(s+3)}$

Solution:

Since it is foster series function $z(s) = \frac{6(s+2)(s+4)}{s(s+3)}$

This is the RC impedance n/w.

Now,

(i) $z(0) = \infty$, C_0 is present .

(ii) $z(\infty) = \sigma$, R_∞ is also present.

$Z(s) = k_o/s + k_\infty + k_1/(s+3) = k_o/s + k_1/(s+3)+6$

$$K_o = \frac{6(s+2)(s+4)}{s(s+3)}.s\bigg|_{s=0} = (6.2.4)/3 = 16$$

$K_2 = 2$

$\therefore Z(s) = 16/s + 2/(s+3) + 6$

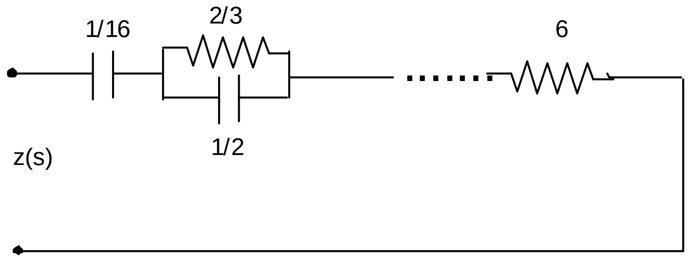
The component values are as follows:

$16/s \Rightarrow 1/c_o s \Rightarrow c_0 = 1/16$ F

$\sigma \Rightarrow R_\infty \Rightarrow R_\infty = 6\ \Omega$

$2/(s+3) \Rightarrow R_1 = 2/3\ \Omega$ and $C_1 = ½$ F

The ckt will be:



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$$F(s) = \frac{6(s+2)(s+4)}{s(s+3)}$$

$$F(s) = z(s) = \frac{6(s+2)(s+4)}{s(s+3)}$$

$= 6+ 16/s + 2/(s+3)$

Forster parallel method for R-C one port n/w:

In this case,

$F(s) = Y(s)$

$Y(s) = k_o/s + k_1/(s+\sigma_1)+ k_2/(s+\sigma_2) + ………+k_\infty$

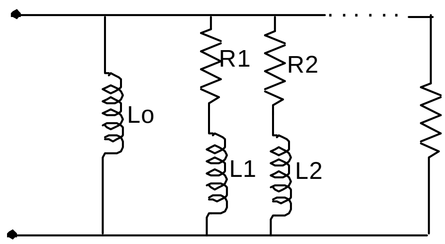


Fig. (i) R-L admittance n/w for foster 2nd method in this case

- k_o/s represents inductor of value $1/k_o$
- k_∞ represents inductor of value $1/k_o$
- $k_i/(s+\sigma_i)$ represents RL series ckt having inductor of value $1/k_i$ H and resister of value σ_i/k Ω .

Properties:

Same as RC- impedance.

Example: 01: Synthesis the following function in foster parallel.

$$F(s) = \frac{6(s+2)(s+4)}{s(s+3)}$$

Solution:

Since it is Foster parallel,

$$F(s) = Y(s) = \frac{6(s+2)(s+4)}{s(s+3)}$$

$$= 6 + 16/s + 2/(s+3)$$

∴ The ckt will be:

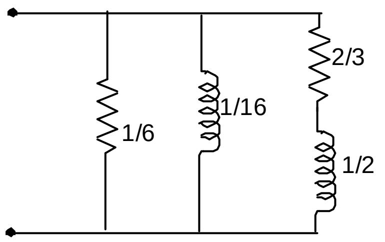


Fig. R-L admittance ckt from foster parallel

Continued Fraction method or cauer method for R-C impedance or R-L Admittance:

1. If F(s) = z(s) , then it yields cauer 1 n/w.
2. If F(s) = Y(s) , then it yields cauer 2 n/w.

For cauer 1 n/w:

In this case F(s) = z(s)

Example:01: Synthesize the following function cauer 1 form.

$$F(s) = \frac{6(s+2)(s+4)}{s(s+3)}$$

Solution:

$$F(s) = z(s) = \frac{6(s+2)(s+4)}{s(s+3)} = \frac{6s^2 + 36s + 48}{s^2 + 3s}$$

Now,

$S^2 + 3s) 6S^2+36s+48 (6$ ⟵ $Z_1(s)$

$\quad 6S^2+18s$

$18s+48) s^2+ 3s \ (s/18$ ⟵ $Y_2(s)$

$\quad S^2+8s/3$

$s/3) 18s+ 4s \ (54$ ⟵ $Z_3(s)$

$\quad 18s$

$48) s/3 \ (s/3.48$ ⟵ $Y_4(s)$

$\quad s/3$

The ckt will be:

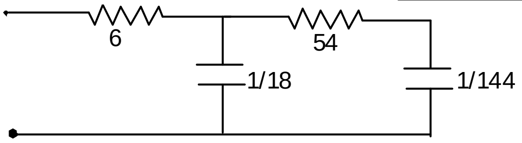


Fig. Caure 1 n/w

Cauer 2 n/w:

Example: 02: Realise the given function in cauer 2 n/w $F(s) = \frac{6(s+2)(s+4)}{s(s+3)}$

Solution:

In this case,

$F(s) = Y(s) = \frac{6(s+2)(s+4)}{s(s+3)}$

In this case circuit will be :

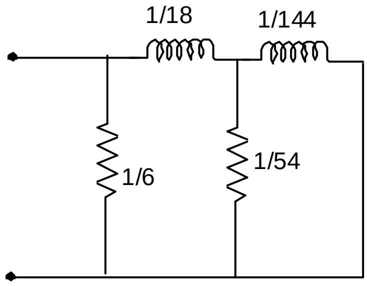
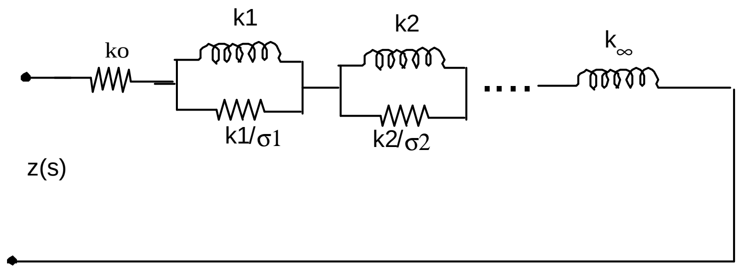


Fig. Caure 2 method

R-L one-Port n/w: (R-L impedance or R-C admittance n/w)

1. **Foster Series method:** It yields R-L impedance ckt for which
   $F(s) = (s) = k_o + k_i s/(s+\sigma_1) + k_2 s/(s+\sigma_2) + \ldots\ldots.. + k_\infty s$



In this case,

- k_0 represent resistor of value $k_o\ \Omega$.
- $k_\infty s$ represent inductor of value k_∞ H.
- $k_i s/(s+\sigma_i)$ represent RL parallel ckt with resistor of value k_i and inductor of value k_i/σ_i .

This method of synthesis is know as foster series (1^{st}) method for R-L one port n/w.

Properties of R-L impedance n/w:

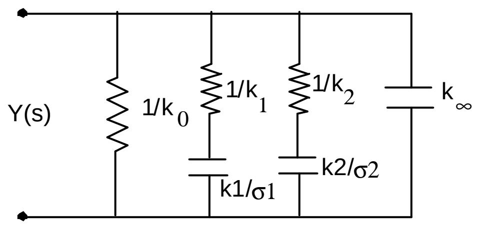
1. Poles are on the –ve real axis.
2. The residue of pole must be real and +ve i.e F(s) must be PRF.
3. $z(0) = k_0$ if R_0 is present.
   $= 0$ if R_0 is missing.
4. $z(\infty) = \infty$ if L is present.
   $= \sum R_i$ if L is missing.
5. $z(\infty) \geq z(0)$
6. Zero is nearest to the origin.
7. The pole and zero must be alternatively placed.

2. Foster parallel method:

In this case,

$F(s) = Y(s) = k_o + k_1 s/(s+\sigma_i) + k_2 s/(s+\sigma_2) + \ldots\ldots\ldots + k_\infty$

The ckt will be as follows:



This method of synthesis is known as Foster parallel method which yields R-C admittance n/w.

Properties:

Some as that of R-L impedance except F(s) = Y(s)

Example:01: Given F(s) = $\frac{4(s+1)(s+3)}{(s+2)(s+6)}$. Realise the above function in (a) Foster series (b) Foster parallel.

Solution:

Since zero is nearest to the origin , (i.e s = -1f) the function yields R-L one port n/w.

(a) Foster series: In this case F(s) = z(s) = $\frac{4(s+1)(s+3)}{(s+2)(s+6)}$

Thus, it yields R-L impedance n/w. To check the availability of components, we use.

$Z(0) = (4\times1\times3)/(2\times6) = 1 = k_o$. i.e R_o is present .

$Z(\infty) = 4 = \sum R_i$, L_∞ is missing.

$$\therefore z(s)/s = \frac{4(s+1)(s+3)}{(s+2)(s+6)} = \frac{1}{s} + \frac{k_1}{s+2} + \frac{k_2}{s+6}$$

$$K_1 = \frac{4(s+1)(s+3)}{s(s+2)(s+6)}.(s+2)\bigg|_{s=-2}$$

$$= \frac{4(-2+1)(-2+3)}{-2(-2+6)}$$

$$= ½$$

$$K_2 = \frac{4(s+1)(s+3)}{s(s+2)(s+6)}.(s+6)\bigg|_{s=-6}$$

$$= \frac{4(-6+1)(-6+3)}{-6(-6+2)}$$

$K_2 = 5/2$

$$\therefore z(s)/s = \frac{1}{s} + \frac{(1/2).s}{s+2} + \frac{(5/2).s}{s+6}$$

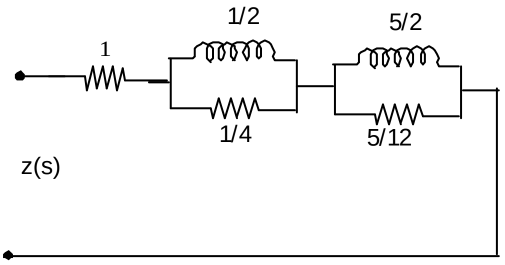


Fig. Foster series n/w

(b) Foster parallel:

In this case,

F(s) = Y(s) = $\frac{4(s+1)(s+3)}{(s+2)(s+6)}$

Which yields R-C admittance n/w.

$\therefore$ Y(s) = $1 + \frac{(1/2).s}{s+2} + \frac{(5/2).s}{s+6}$

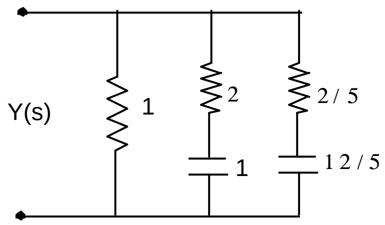


Fig. Foster Parallel ckt.

Cauer Method for R-L one port n/w:

(1) If F(s) = z(s) , it is called cauer 1 method which yields R-L impedance ckt.
(2) If F(s) = Y(s) , it is called caure 2 method which yields R-C admittance ckt.

Example: 01: Synthesize the following function in

(a) caure 1 n/w. (b) cauer 2 n/w. $\frac{4s^2 + 16s + 12}{s^2 + 8s + 12}$

Solution:

(a) cauer 1 n/w:

In this case

F(s) = z(s) = $\frac{4(s+1)(s+3)}{(s+2)(s+6)} = \frac{4s^2 + 16s + 12}{s^2 + 8s + 12}$

S^2+8s+12) $4s^2$+ 16s+12 (4
$4s^2$+ 32s+4s
-ve

This way the ckt cannot be realize. Therefore z(s) is rewritten in form as:

Z(s) = $\frac{12 + 16s + 4s^2}{12 + 8s + s^2}$

$12+8s+s^2 \,)\, 12+16s+4s^2 \,(\, 1 \leftarrow Z_1(s)$

$\quad 12+8s+s^2$

$\quad 8s+3s^2 \,)\, 12+8s+s^2 \,(\, 3/2s \leftarrow Y_2(s)$

$\quad\quad 12+9s/2$

$\quad\quad 7s/2+s^2 \,)\, 8s+3s^2 \,(\, 16/7 \leftarrow Z_3(s)$

$\quad\quad\quad 8s+16s^2/7$

$\quad\quad\quad 5s^2/7 \,)\, 7s/2+s^2 \,(\, 49/10s \leftarrow Y_4(s)$

$\quad\quad\quad\quad 7s/2$

$\quad\quad\quad\quad s^2 \,)\, 5s^2/7 \,(\, 5/7 \leftarrow Z_5(s)$

$\quad\quad\quad\quad\quad 5s^2/7$

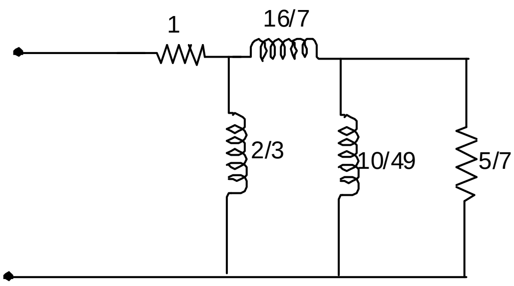


Fig. cauer 1 n/w

(b) Cauer 2 n/w:

In this case,

$$F(s) = Y(s) = \frac{4s^2+16s+12}{s^2+8s+12} = \frac{12+16s+4s^2}{12+8s+s^2}$$

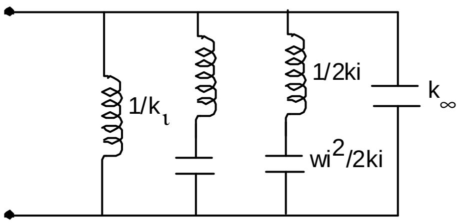


Fig. Cauer 2 n/w

Assignment: 03

1. $F(s) = \frac{(s+2)(s+4)}{(s+1)(s+3)}$ Find the n/w of the form (a) Foster series (b) Foster parallel.
2. Realize the n/w function $F(s) = \frac{(s+1)(s+3)}{(s+2)(s+4)}$ (a) 1st Foster method. (b) 2nd foster method.
3. Realise the n/w function $Y(s) = \frac{(s+2)(s+4)}{(s+1)(s+3)}$ as a cauer n/w.
4. $z(s) = \frac{(s+1)(s+3)}{(s+2)(s+2)}$ Realise the function in foster and cauer n/w.

5. Realise the n/w $Y(s) = \frac{(s+2)(s+4)}{(s+1)(s+6)}$

Two port n/w:

1. Z-Parameter
2. Y – Parameter
3. ABCD Parameter
4. Transformation of one parameter to other
5. T and π n/w
6. Interconnection of two port n/w
   a. Cascade b. series c. parallel.

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Chapter: 4

Low pass Filter Approximations:

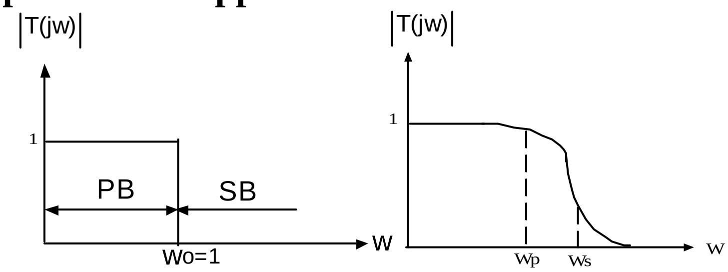

Fig. (a) Ideal case (b) Non ideal case

The desirable feature of low pass approximation are

1. Minimum pass band attenuation, α_p
2. Maximum stop band attenuation, α_s
3. Low transition band ratio, w_s/w_p
4. Simple network.

The approximation Method are:

1. Butterworth
2. Chebyshev
3. Inverse chebyshev
4. Ellipse or Cauer
5. Bessel –Thomson

1. **Butterworth low pass approximation:** Generally signal become contaminated with high frequency signal. It is evident that low pass filter are required to remove such unwanted signals from the useful one. The desirable LPF response is shown in fig . 1(a)

Below the normalize frequency i.e $w_0 = 1$, the amplitude $|T(jw)|$ is constant and above this frequency it is zero. Pass band and stop band are clearly separated at $w_o = 1$. But since the ideal response can not be achieve . We make the approximation based on the ideal response.

We make the magnitude T(jw) nearly constant in PB. In the SB, we require sharp roll – off (n-pole roll –off). Where 'n ' will be large no if abrupt transition from PB to SB is desired.

Mathematically, we can write,

$T(jw) = Re[\, T(jw)\,] + j\, I_m[\, T(jw)]$

Re[T(jw)] = Real part of T(jw)
I_m [T(jw)] = Imaginary part of T(jw).
Where it is to be noted that Re[T(jw)] indicates an even functions.
Where I_m[T(jw)] indicates it is an odd function.
Agains,
T*(jw) = T(-jw) = Re[T(jw)]+jI_m [T(jw)] ……….(ii)The functions so obtained is called conjugate of T(jw)
Thus (i) and (ii) gives
T(jw) T*(jw) = $|T(jw)|^2$ = $Re[T(jw)]^2 + jI_m[T(jw)]^2$ ……(iii)
T(jw) T*(jw) = T(s) T*(s) = $|T(s)|^2$
The function $|T(s)|^2$ (or $|T(jw)|^2$) is called magnitude squared function.

Example 01: Find the magnitude square function for
$T(s) = (s+2) / (s^3 + 2s^2 + 2s+3)$
$T(s) = -s+2 / -s^3 + 2s^2 – 2s +3$
$\therefore |T(s)|^2$ = T(s) . T(-s)
$= (2+s)/(s^3+2s^2 +2s+3) \times (2-s)/(-s^3 +2s^2 – 2s+3)$
= ………..

The magnitude square function is an even function which can be represented by using a numerator and denominator polynomial that are both even, i.e

$$|T(jw)|^2 = \frac{A(w^2)}{B(w^2)}$$

$$|T(jw)|^2 = \frac{A_0 + A_2 w^2 + A_4 w^4 + ... + A_{2n} w^{2n}}{B_0 + B_2 w^2 + B_4 w^4 + ... + B_{2n} w^{2n}}$$

$$|T(jw)|^2 = \frac{A_0}{B_0 + B_2 w^2 + B_4 w^4 + ... + B_{2n} w^{2n}}$$

Here $A_2 = A_4 = ……A_{2n} = 0$ (assumption).
The choice has been made as per our inspection on the roll off that was directly dependent on the number of poles. This means larger the difference between degree of A and B , we get the larger roll-off . This will give us a direct n-pole roll off for T_n(jw) or T_n(s) which will be know as “ All pole” function.

Special case:
We assume ,
$B_2 = B_4 = 0$
$B_{2n} = (1/w_0)^{2n}. B_0$ and $A_0 = B_0$
Now , putting these assumption in the equation (i) we get,

$$|T(jw)|^2 = \frac{A_o}{B_0 + B_{2n} w^{2n}}$$

$$= \frac{B_o}{B_0 + \left(\frac{1}{w_0}\right)^{2n} B_0}$$

$$= \frac{1}{1+\left(\frac{1}{w_0}\right)^{2n} w^{2n}}$$

$$|T(jw)|^2 = \frac{1}{1+\left(\frac{w}{w_0}\right)^{2n}} \quad \ldots\ldots\ldots\ldots\text{(ii)}$$

In generalize condition,

$w_o = 1$

$$|T(jw)|^2 = \frac{1}{1+(w)^{2n}} \quad \ldots\ldots\ldots\ldots\text{(iii)}$$

$$|T(jw)|^2 = \frac{1}{\sqrt{1+(w)^{2n}}} \quad \ldots\ldots\ldots\ldots\text{(iv)}$$

From equation (iv) the following property can be written.

1. At w = 0 , i.e T(j0) = 1 for all values of n.
2. At w = 1 (=w_0), i.e T(j1) = 0.707 for all values of n.
3. At w = ∞ , i.e T(j ∞)= 0 for all value of n.
4. For large values of w; T_n(jw) exhibits larger roll off.
5. Butterworth response , also known as, maximally flat response, is all pole functions.
6. Butterworth (BU) response can be expanded in Taylor's series from as:

$$|T(jw)|^2 = \frac{1}{\sqrt{1+(w)^{2n}}}$$

$= (1+w^{2n})^{-1/2}$

$= 1+ ½ . w^{2n} + (1/2)^2. (w^{2n})^2/2! - \ldots\ldots\ldots..$

$\approx 1 – ½. w^{2n}$

∴ In Taylor series,

$$\boxed{|T(jw)| = \left(1-\frac{1}{2}w^{2n}\right)} \quad \ldots\ldots\ldots..\text{(v)}$$

Again we know ,

$$|T(jw)|^2 = \frac{1}{1+(w)^{2n}}$$

Putting jw = s

$$|T(s)|^2 = \frac{1}{1+\left(\frac{s}{j}\right)^{2n}} = \frac{1}{1+\frac{s^{2n}}{j^{2n}}} = \frac{1}{\frac{1+s^{2n}}{(-1)^n}} = \frac{1}{1+(-1)^n s^{2n}}$$

$$\boxed{T(s)|^2 = \frac{1}{1+(-1)^n s^{2n}}} \quad \ldots\ldots\ldots\text{(vi)}$$

Which gives the butterworth response in s-domain

Evaluation of T(s) for BU – Response:

(i) For n = 1 equation (vi) becomes

$|T(s)|^2 = \frac{1}{1-s^2}$

$s^2 = 1$

$s = \pm 1$

$\therefore |T(s)|^2 = 1/(1-s)(1+s)$

$= 1/(1+s).\ 1/(1-s)$

$= T(s)\ .\ T(-s)$

$T(s) = 1/(s+1)$

NOTE:

(i) If $s^n = -1$, then, $s = 1\angle (180+k360)/n$, k = 0, 1…..(n-1) in s domain.

(ii) If $s^n = -1$, then, $S = 1 \angle k360/n$, k =0,1, 2….(n-1)

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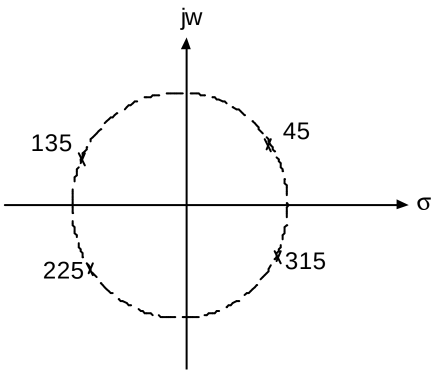
Butterworth transfer function (continued ……)

(ii) For n = 2

Equation (vi) becomes ;

$|T(s)|^2 = \frac{1}{1+(-1)^2 s^4}$



$= \frac{1}{1+s^4}$

To get the poles ,

$1+s^4 = 0$

$S^4 = -1$

$S = 1 \angle (180^\circ + k360^\circ)/4$, k = 0, 1, 2, 3 [since n = 4]

$S = 1 \angle 45^\circ, 135^\circ, 225^\circ, 315^\circ$

The poles that lie on the left half of s-plane are:

$S = 1 \angle 135^\circ, 225^\circ$

Or $S = -0.0707 \pm j0.707 = s_1, s_2$

$\therefore T(s) = \frac{1}{(s-s_1)(s-s_2)}$

$= \frac{1}{(s+0.707-j0.707)(s+0.707+j0.707)}$

$= \frac{1}{s^2+\sqrt{2}s+1}$

(iii) For n = 3

$$|T(s)| = \frac{1}{1+(-1)^3 s^6}$$

$$= \frac{1}{1-s^6}$$

To get the pole

$1-s^6 = 0$

$S^6 = 1$

$S = 1 \angle k360/n$, $k = 0,1,2 \ldots\ldots\ldots(2n-1)$

$S = 1\angle 0, 60, 120, 180, 240, 300$

The poles that lie on left half of s-plane are

$S = 1 \angle 120, 180, 240$

Or, $= 1\angle 120, 1\angle 180, 1\angle 240$

$S_1 = -0.5 + j0.866$

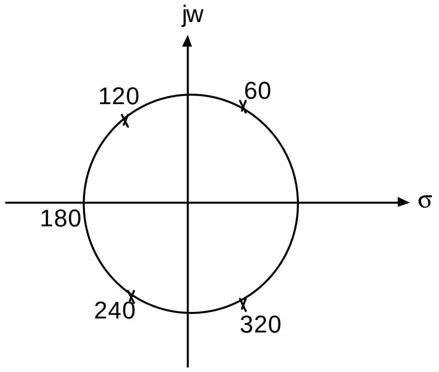
$S_2 = -1+j0$

$S_3 = -0.5 - 0.866j$

$$\therefore \ |T(s)| = \frac{1}{(s-s_1)(s-s_2)(s-s_3)}$$

$$= \frac{1}{(s+1)(s-0.5-0.866j)(s+0.5-0.866j)}$$

$$= \frac{1}{(s+1)(s^2+s+1)}$$



Order and cutoff frequency for Butterworth:

It is to noted that, at w =w_p , $\alpha = \alpha_p = \alpha_{max}$

And at w = w_s , $\alpha = \alpha_s = \alpha_{min}$

We know that

$$|T(s)|^2 = \frac{1}{1+\left(\frac{w}{w_o}\right)^{2n}}$$

Also the attenuation formula is given by ;

$\alpha = -20\log |T(s)|$

$$\alpha = -20\log_{10}\left\|\left[\frac{1}{1+\left(\frac{w}{w_o}\right)^{2n}}\right]\right\|$$

$$\alpha = -20\log_{10}\left|1+\left(\frac{w}{w_o}\right)^{2n}\right|^{-\frac{1}{2}}$$

$$\alpha = 10\log_{10}\left|1+\left(\frac{w}{w_o}\right)^{2n}\right| \quad \text{……………….(i)}$$

$$\alpha/10 = \log10\left|1+\left(\frac{w}{w_o}\right)^{2n}\right|$$

$$10^{\alpha/10} = 1+\left(\frac{w}{w_o}\right)^{2n}$$

$$\left(\frac{w}{w_o}\right)^{2n} = 10^{\alpha/10} - 1$$

$$\left(\frac{w}{w_o}\right) = (10^{\alpha/10} - 1)^{1/2n}$$

$$w = \frac{w}{(10^{\alpha/10} - 1)^{\frac{1}{2n}}}$$

Now at $w = w_p$, $\alpha = \alpha_{max}$

$$w_o = \frac{w_p}{(10^{\alpha \max/10} - 1)^{\frac{1}{2n}}} \quad \text{…………(ii)}$$

and at $w = w_s$, $\alpha = \alpha_{min}$

$$w_o = \frac{w_s}{(10^{\alpha \min/10} - 1)^{\frac{1}{2n}}} \quad \text{………………..(iii)}$$

equating (i) and (ii) can be equated as:

$$\frac{w_p}{(10^{\alpha \max/10} - 1)^{\frac{1}{2n}}} = \frac{w_s}{(10^{\alpha \min/10} - 1)^{\frac{1}{2n}}}$$

$$\frac{w_p}{w_o} = \frac{(10^{\alpha \max/10} - 1)^{\frac{1}{2n}}}{(10^{\alpha \min/10} - 1)^{\frac{1}{2n}}}$$

$$\left(\frac{w_p}{w_o}\right)^{2n} = \frac{(10^{\alpha \max/10} - 1)}{(10^{\alpha \min/10} - 1)}$$

Taking log on both sides,

$$20 \log\left(\frac{w_p}{w_o}\right)^{2n} = \log \frac{(10^{\alpha \max/10} - 1)}{(10^{\alpha \min/10} - 1)}$$

$$n = \log \frac{(10^{\alpha \max/10} - 1)}{(10^{\alpha \min/10} - 1)} / 2 \log\left(\frac{w_p}{w_o}\right)$$

Now let us find expression for transition band ratio , i.e
TBR = w_s/w_p , where , TBR = Transition band ratio.
$W_s/w_p = [(10^{\alpha\ min/10} - 1)/(10^{\alpha\ max/10}-1)]^{1/2n}$ ………….(v)

Example 01: Consider a filter using a butterworth response to realize the following specifications of LPF.
$\alpha_{max} = 0.5$ dB
$\alpha_{min} = 20$ dB
$w_p = 1000$ rad/sec
$w_s = 2000$ rad/sec
Determine the order and cut off frequency for the filter.
Solution:
$n = 4.83 \approx 5$
$w_o = 1234.12$ rad/sec
Note: Always choose higher value of 'n' (i.e the order of filter)because it provides larger roll off which decreases attenuation.

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2. Chebyshev Approximation Method For LPF :

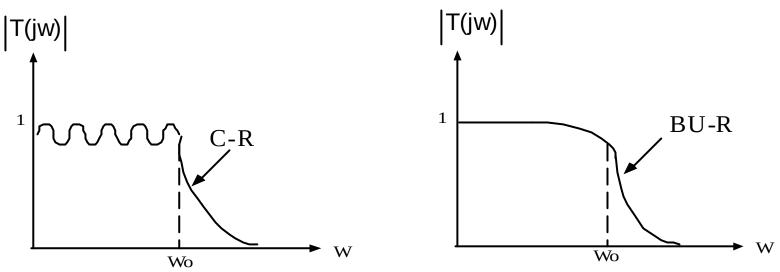


Fig (i) (a) Chebyshev response (b) butterworth response

The generalize low pass filter can be represented by

$$|T_n(jw)|^2 = \frac{1}{1+[F_n(w)]^2} \quad \text{……………….(i)}$$

For Butterworth
$F_n(w) = (w/w_o)^n$
With $w_0 = 1$
$F_n(w) = w^n$
Similarly to butterworth we have to determine the function $F_n(w)$ for chebyshev response for which the concept of Lissagious figure is required.

Lissagious figure:

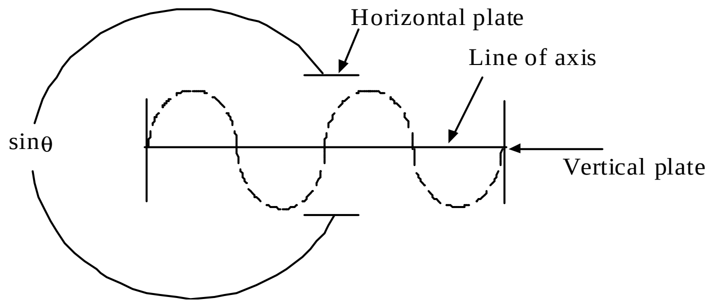
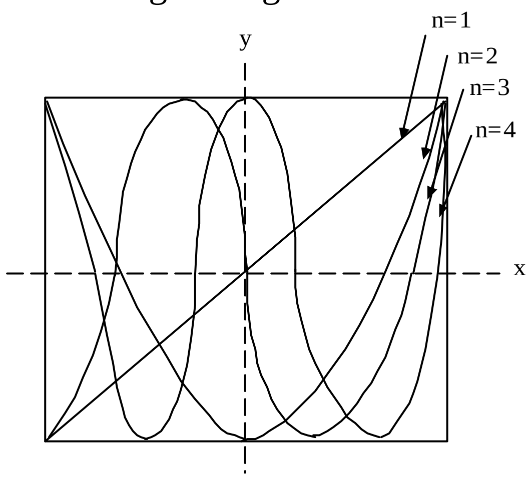


Fig (ii) (a) CRO Lissagious figure.



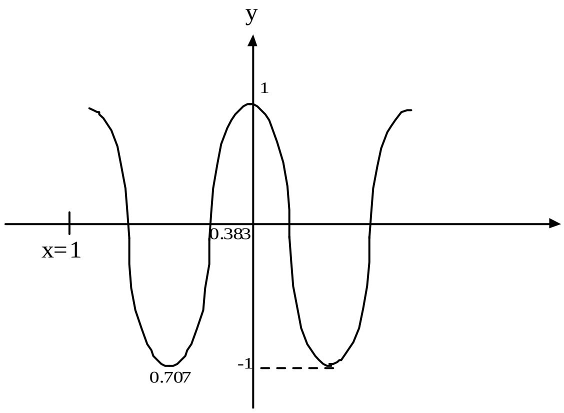
Fig(ii) (b) Lissagious figure for n = 1,2,3 and 4

When adjustable frequency multiple of fixed frequency is applied , stationary figures are obtained which are know as Lissagious figures.

Analysis:
Let the deflection due to voltage on horizontal plates be
$x = \cos kT$ …………….(ii)
Where , $k = 2\pi/T$
The deflection due to voltage on vertical plates will be then,
$y = \cos nkT$ …………….(iii) Where n is integer and proves the multiple frequencies.
From (ii),
$KT = \cos^{-1}x$
$y = \cos n \cos^{-1}x$ ……………..(iv)
$c_n(x) = \cos n \cos^{-1}x$ which is the equation for Lissagious figures.

Example: If n = 4
Assume, $\theta = \cos^{-1}x$
$x = \cos\theta$
Then,
$y = \cos 4\theta$

θ	**x**	**4θ**	**y**
0	1	0	1
22.5	0.924	90	0
45	0.707	180	-1
67.5	0.383	270	0
90	0	360	1



Analyse the same for n = 3 and 5.

Chebyshev magnitude Response:

We know that ,

$$|T_n(jw)|^2 = \frac{1}{1+[F_n(w)]^2}$$

Where $F_n(w) = \in c_n(w)$; $\in \leq 1$

Where $c_n(w) = \cos n \cos^{-1} w$

Therefore the magnitude square response will be

$$|T_n(jw)|^2 = \frac{1}{1+\in^2 c_n^{\,2}(w)} \quad \ldots\ldots\ldots\ldots\ldots\text{(vi)}$$

This function (i.e $c_n(w)$) is valid within the range $w = \pm 1$. However , the function must also be valid for longer value of w for which we should refine our assumption for $c_n(w)$.

$\therefore$ w > 1,

Let,

$\cos^{-1}(w) = jz$

w = cosjz

we know that ,

$$\cos jz = \frac{e^{j(jz)} + e^{-j(jz)}}{2} = \frac{e^{z} + e^{-z}}{2} = \cosh z$$

$\therefore$ cosjz = coshz

$\therefore$ w = coshz

Z = $\cosh^{-1}w$

$\therefore$ w = cosj $\cosh^{-1}w$

$\therefore \cos^{-1}(w) = j\cosh^{-1}w$

$\therefore c_n(w) = \cos n \cos^{-1}w$

$= \cos nj \cosh^{-1}w$

$= \cos j(n\cosh^{-1}w)$

$= \cosh n \cosh^{-1}w$

$\therefore$

$c_n(w) = \cosh \cosh^{-1}w$, w> 1 $C_n(w) = \cos n \cos^{-1}w$, $w = \pm 1$

Properties of magnitude response for Chebyshev:

We know that,

$$|T_n(jw)|^2 = \frac{1}{1+\in^2 c_n^2(w)}$$

$$|T_n(jw)| = \frac{1}{\sqrt{1+\in^2 c_n^2(w)}}$$

Where, $c_n(w) = \cos n \cos^{-1} w \qquad w \le 1$

$= \cosh n \cosh^{-1} w \qquad w \ge 1$ and $|\in| \le 1$

1. At w = 0,
   $C_n(0) = \cos n\pi/2$; 0,1,2…….
   $|T_n(jw)| = 1$ for n = odd
   $= \frac{1}{\sqrt{1+\in^2}}$ for n = even

2. w = 1
   $c_n(1) = 1$ for all values of n.
   $\therefore |T_n(jw)| = \frac{1}{\sqrt{1+\in^2}}$

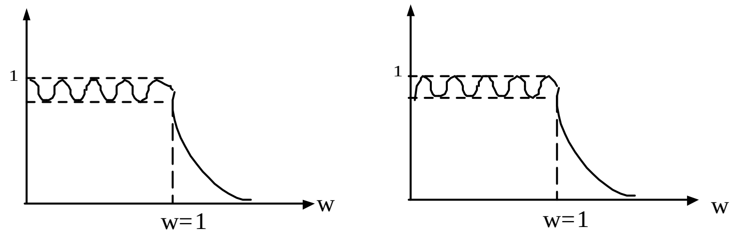


Fig (iii) (a) C-R for n = odd (b) C-R for n = even

Order of C-R filter:

We know , the attenuation formula is given by

$\alpha = -20\log |T_n(jw)|$ dB

But, $|T_n(jw)| = \frac{1}{\sqrt{1+\in^2 c_n^2(w)}} = \left(\frac{1}{1+\in^2 c_n^2(w)}\right)^{\frac{1}{2}}$

$$\therefore \alpha = -20\log \left|\left(\frac{1}{1+\in^2 c_n^2(w)}\right)^{\frac{1}{2}}\right|$$

$$= -10\log\left|\frac{1}{1+\in^2 c_n^2(w)}\right|$$

$\alpha = 10\log |1+\in^2 c_n^2(w)|$ …………(vii)

$\therefore \alpha = 10\log |1+\in^2 (\cos n\cos^{-1} w)^2| \qquad |w| \le 1$

for $|w| > 1$,

$\alpha = 10\log |1+\in^2 (\cosh n\cosh^{-1} w)^2|$ ....(ix)

Now ,

α_{max} occurs when , $c_n(w) = 1$

$\therefore$ equation (vii) reduces to ,
$\alpha = \alpha_{max} = 10 \log(1+\in^2.1)$………(x)
$\alpha_{max} / 10 = \log(1+\in^2.1)$
$1+\in^2 = 10^{\alpha\ max/10}$

$$\boxed{\in = (10^{\alpha \max/10} - 1)^{\frac{1}{2}}}$$ …………….(xi)

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Here
we know that
$w = w_{np}$, then, $\in^2 c_n^2(w) = 1$

$c_n(w_{np}) = \frac{1}{\in} = \cosh(n\cosh^{-1} w_{np})$ [since $w_{np} > 1$]

$\text{Cosh}^{-1}(n\cosh^{-1} whp) = \frac{1}{\in}$

$\text{Cosh}^{-1}(n\cosh^{-1} whp) = \frac{1}{\in}$

$\text{Cosh}^{-1} whp = 1/n. \cosh^{-1}(\frac{1}{\in})$

$\therefore w_{np} = \cosh(1/n. \cosh^{-1}(\frac{1}{\in}))$ ………(xii)

$W_{np} = \cosh[1/n. \cosh^{-1}(\{10^{\alpha max/10} - 1\}^{1/2})]$
Now $\alpha = \alpha_{min}$ when $w = w_s$
$\therefore \alpha_{min} = 10 \log_{10}(1+\in^2 c_n^2(w_s))$
$\in^2 c_n^2(w_s) = 10^{\alpha\ min/10} - 1$
$\varepsilon^2(\cosh n\cos^{-1} ws)^2 = 10^{\alpha\ min/10} - 1$
Or, $(\cosh n\cosh^{-1} w_s)^2 = (10^{\alpha\ min/10} - 1)/(10^{\alpha\ max/10} - 1)$
$n \cosh^{-1} w_s = \cosh^{-1}[(10^{\alpha\ min/10} - 1)/(10^{\alpha\ max/10} - 1)]^{1/2}$
$\therefore n = \{\cosh^{-1}[(10^{\alpha\ min/10} - 1)/(10^{\alpha\ max/10} - 1)]^{1/2}\}/\cosh^{-1} w_s$ …..(xiii)

Example: Given $w_p = 1$, $w_s = 2.33$, $\alpha_{max} = 0.5$dB , $\alpha_{min} = 22$ dB. Calculate 'n' for Butterworth and chebyshev filters which filter would you select.
Solution: For Butterworth filter , the order is given by
$n = \log_{10}[(10^{\alpha\ max/10} - 1)/(10^{\alpha\ min/10} - 1)]/ 2\log(w_p/w_s)$
$= \log[(10^{0.5/10} - 1)/(10^{22/10} - 1)]/2\log(1/2.33)$
$= 4.234 \approx 5$

$\therefore$ n for BU = 5
For Chebyshev the order is given by ,
$n = \cosh^{-1}[(10^{\alpha min/10} - 1)/(10^{\alpha max/10} - 1)]/\cosh^{-1}(2.33)$
$= 2.89 \approx 3$

n for chebyshev = 3 .

Since the order of chebyshev filter (i.e n =3) is less then the order of butterworth filter (i.e n = 5) and both filter provides the same roll- off for the specification, n would choose chebyshev filter.

Chebyshev poles location and network function:

We know

$|T(jw)|^2 = \frac{1}{1+\in^2 c_n^2(w)}$ ……..(i)

Substituting s = jw equation (i) becomes,

$|T(s)|^2 = \frac{1}{1+\in^2 c_n^2(s/j)}$ ………..(ii)

To determine the poles,

$1+\in^2 c_n^2(s/j) = 0$

$c_n(s/j) = \pm j\frac{1}{\in}$ ……(iii)

Again,

C_n (s/j) = cosn $\cos^{-1}$(s/j)

Let

Cos^{-1}(s/j) = x = u + jv

Then, c_n(s/j) = cosnx = cosn (u+jv)

= cosnu. Cosnjv – sinnu. Sin njv

= cosnu coshnv – jsin nu . sinh nv

= 0 $\pm j\frac{1}{\in}$ [from equ. (iii)]

Thus, comparing , we get, [cosjnv = coshnv

Cosnu . cosh nv = 0 [sinjnv = jsinhv]

-sinnu. Sinhnv = 0

∴ The minimum value of

Coshnv = 1, coshnv not equal to 0

∴ cosnu = 0

Or cosnu_k = cos(2k+1). Π /2, k = 0,1,2………….

U_k = (2k+1) π/2n ……….(v)

Now ,

-sinnu_k = sinhnv_k = $\pm\frac{1}{\in}$

But, sin nu_k = +- 1

∴ +-1 . sinhnv_k = $\pm\frac{1}{\in}$

Or sinhnv_k = $\frac{1}{\in}$

Nv_k= $\sinh^{-1}(\frac{1}{\in})$

V_k = 1/n. $\sinh^{-1}(\frac{1}{\in})$

Again, we know that

Cos^{-1} (s/j) = x = u +jv

s/j = cosx = cos(u+jv)

in general,

$s_k = j\cos(u_k + jv)$

$= j[\cos u_k .\cos jv - \sin u_k . \sin jv]$

$= j[\cos u_k . \cosh v - j\sin u_k . \sinh v]$

$S_k = \sin u_k . \sinh v + j\cos u_k . \cosh v$ ……(vi) , k = 0,1,2….(2n-1)

Again,

$S_k = \sin[(2k+1) \pi /2n] \sinh v + j\cos[(2k+1)\pi /2n] \cosh v$

Or , $s_k = \sigma_k + jw_k$ ………..(viii)

Where,

$\Sigma_k = \sin[(2k+1) \pi /2n] \sinh v$ ……….(ix)

$W_k = \cos[(2k+1)\pi /2n] \cosh v$ ………..(x)

Form euation (ix) ,

$\sigma_k^2/\sin^2 hv = \sin^2[(2k+1)\pi /2n]$ ……..(xi)

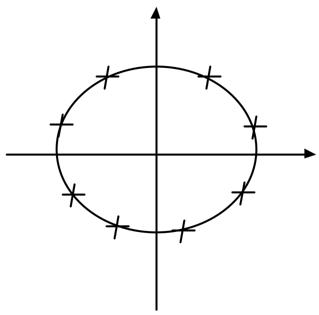
and from quation (x)

$w^2k/\cos^2 hv = \cos^2 [(2k+1)\pi /2n]$ ……..(xii)

Now adding equation (xi) and (xii) we get,

$\sigma_k^2/\sin^2 hv + w^2k/\cos^2 hv = 1$ ……….(xii)

Which is equation of ellipse . Therefore we can say that the poles of chebyshev filter lie on the ellipse.



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Example:01 Obtained the 4th order network function of a low pass chebyshev filter with α_{max} = 0.75 dB

Solution: n = 4 α_{max} = 0.75 dB

Now $\varepsilon = (10^{\alpha max/10} -1)^{1/2}$ $w_{hp} = \cosh (1/n. \cosh^{-1} (1/\varepsilon))$

$= (10^{0.75/10} -1)^{1/2} = 0.434$

And $w_{hp} = \cosh (1/n. \cosh^{-1}(1/\varepsilon))$ =

Pole location is given by

$S_k = \sin u_k \sinh v + j\cos u_k \cosh v$
Where, $u_k = (2k+1) .\pi/2n$; $k = 0, 1, \ldots\ldots\ldots 2n-1$
$V = 1/n. \sinh^{-1}(1/\varepsilon)$

$\therefore u_o = \pi/8$ $u_1 = 3\pi/8$, $u_2 = 5\pi/8$, $u_3 = 7\pi/8$, $u_4 = 9\pi/8$
$u_5 = 11\pi/8$ $u_6 = 13\pi/8$, $u_7 = 15\pi//8$
v = 0.393 (adjust calculator in radian)

$s_0 = 0.154 + 0.996j$

$s_1 = 0.373+ 0.413j$

$s_2 = 0.373 – 0.413j$

$s_3 = 0.154-0.996j$

$s_4 = -0.154 – 0.996j$

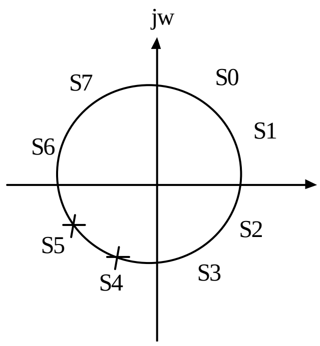
$s_5 = -0.373 – 0.413j$

$s_6 = -0.373 + 0.413j$

$s_7 = -0.154 + 0.996j$

The transfer function (or n/w function) for forth order chebyshev filter is given by ,

$T(s) = 1/(s+s_4)(s+s_5)(s+s_6)(s+s_7)$



Home Assignment:
Example:02: Determine the network function for 3rd order chebyshev LPF with $\alpha_{max} = 0.75$ dB ($=\alpha_p$; pass band attenuation)

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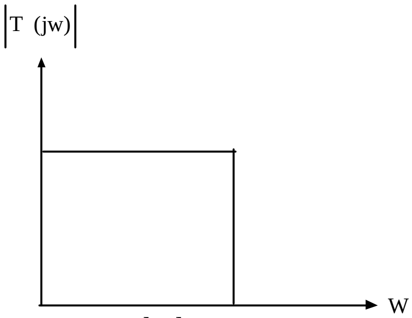
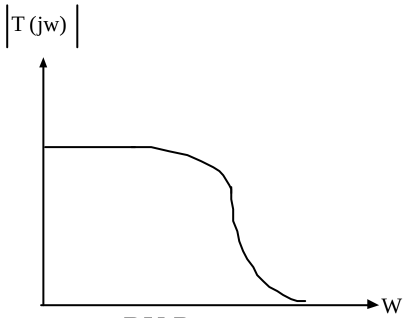
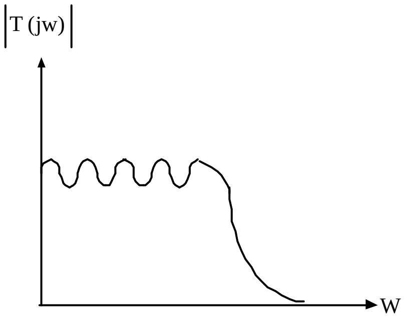
Inverse chebyshev low pass approximation:



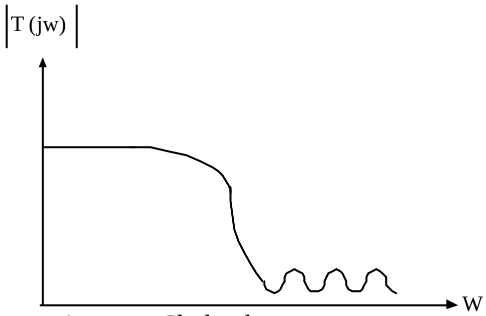
Ideal LPF



BU-Response



Chebyshev-respone



inverse-Chebyshev-response

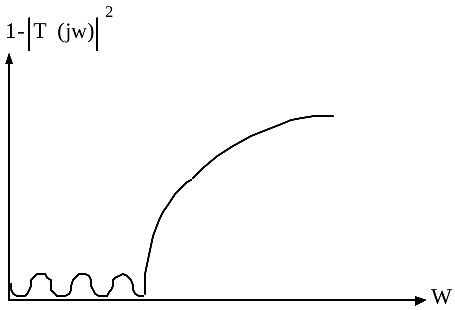


Fig: intermediate stage to obtain inverse chebyshev response.

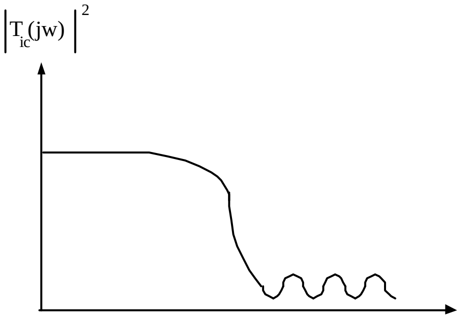


Fig: The reciprocal value of w of intermediate stage give the value of w in I-C response.

We know the response of chebyshev is given by

$$|T(jw)|^2 = \frac{1}{1+\in^2 c_n^2(w)}$$

$$1-|T_c(jw)|^2 = 1- \frac{1}{1+\in^2 c_n^2(w)}$$

$$= \frac{\in^2 c_n^2(w)}{1+\in^2 c_n^2(w)}$$

Now replace w by 1/w

$|T_{IC}(jw)|^2 = \frac{\in^2 c_n^{\,2}(1/w)}{1+\in^2 c_n^{\,2}(1/w)}$ ………………(i)

Where,

$|T_{IC}(jw)|^2$ is the magnitude square response for I-C.

We know ,

$c_n(1/w) = \cos n\cos^{-1}(1/w)$

at for w = 1

$c_n(1) = 1$ for all value of n

Thus equation (i) becomes

$|T_{IC}(j.1)|^2 = \frac{\in^2 .1}{1+\in^2}$

$|T_{IC}(j.1)| = \sqrt{\frac{\in^2 .1}{1+\in^2}}$ ……. (ii)

We know that ,

$\alpha_{min} = -20\log|T_{IC}(j.1)|$ dB ……(iii)

Using equation (ii) on equation (iii) , we get,

$\alpha = \alpha_{min} = -20\log|T_{IC}(j.1)|$ dB

$= -20\log\left|\left(\frac{\in^2}{1+\in^2}\right)^{1/2}\right|$

$= 10\log\left|\left(\frac{1+\in^2}{\in^2}\right)\right|$

$\therefore \alpha_{min} = 10\log [\, 1+\frac{1}{\in^2} \}$

Or , $10^{\alpha min/10} - 1 = \frac{1}{\in^2}$

$\in = \left(10^{\alpha\min/10} - 10\right)^{-\frac{1}{2}}$ …………. (iv)

Again in general, the attenuation formula can be written as:

$\alpha = -10\log\left[\frac{\in^2 c_n^{\,2}(1/w)}{1+\in^2 c_n^{\,2}(1/w)}\right]$

$\alpha = 10\log\left[1+\frac{1}{\in^2 c_n^{\,2}(1/w)}\right]$

Now at $w = w_p$ $\alpha = \alpha_{max}$

Then above equation becomes

$\alpha = \alpha_{max} = 10\log\left[1+\frac{1}{\in^2 c_n^{\,2}(1/w_p)}\right]$

$(10^{\alpha\, max/10} - 1) = \frac{1}{\in^2 c_n^{\,2}(1/w_p)}$

$c_n^{\,2}(1/w_p) = \frac{1}{\in^2}.\frac{1}{(10^{\alpha\max/10} - 1)}$

$$c_n^{\,2}(1/w_p) = \frac{(10^{\alpha\min/10}-1)}{(10^{\alpha\max/10}-1)}$$

$$c_n(1/w_p) = \sqrt{\frac{(10^{\alpha\min/10}-1)}{(10^{\alpha\max/10}-1)}} \quad \ldots\ldots\ldots.(v)$$

$c_n(1/w_p)$ = coshn $\cosh^{-1}$ $(1/w_p)$ …………. (vi)

[$\therefore$ $w_p < 1$, $1/w_p > 1$]

Thus equating equation (v) and (vii)

$$\text{Coshn}\cosh^{-1}(1/w_p) = \left[\frac{(10^{\alpha\min/10}-1)}{(10^{\alpha\max/10}-1)}\right]^{\frac{1}{2}}$$

$$\therefore\ n = \frac{\cosh^{-1}\left[\frac{(10^{\alpha\min/10}-1)}{(10^{\alpha\max/10}-1)}\right]^{\frac{1}{2}}}{\cosh^{-1}(1/w_p)} \quad \ldots\ldots\ldots\ldots\ldots\ldots(vii)$$

Which gives the required order for the inverse chebyshev filter.

Now , for half power frequency i.e at $w = w_p$

$|T_{IC}(j.1)| = 1/\sqrt{2}$

$|T_{IC}(j.1)|^2 = ½$

Which means,

$\in^2 c_n^{\,2}(1/w_P) = 1$

$$c_n^{\,2}(1/w_{np}) = \frac{1}{\in^2}$$

$$c_n(1/w) = \frac{1}{\in}$$

$$\text{Coshn}.\cosh^{-1}(1/w_{np}) = \cosh^{-1}\frac{1}{\in}$$

$$n\ \text{coshn}.\cosh^{-1}((1/w_{np}) = \cosh^{-1}(\frac{1}{\in})$$

$$\cosh^{-1}((1/w_{np}) = 1/n.\ \cosh^{-1}(\frac{1}{\in})$$

$$1/w_{np} = \cosh[1/n.\ \cosh^{-1}(\frac{1}{\in})]$$

$$w_{np} = \frac{1}{\cosh\left[\frac{1}{n}\cosh^{-1}(\frac{1}{\in})\right]} \quad < 1 \quad \ldots\ldots\ldots.(viii)$$

Which gives the desire half power frequency.

Example: 01

Given, α_{max} = 0.5 dB

α_{min} = 22 dB

w_p = 0.9

n = ?

w_{np} = ?

Assignment:

Example:02 Differentiate between Butterworth , chevyshev and inverse chebysehev filters.

Pole zero location for inverse chebyshev:

We know that ,

$$|T_{IC}(jw)|^2 = \frac{\in^2 c_n^{\,2}(1/w)}{1+\in^2 c_n^{\,2}(1/w)}$$

T(s). T(-s) = z(s).z(-s)/[p(s).p(-s)]

Where, z(s) z(-s) $|_{s = jw}$ = $\in^2 c_n^{\,2}(1/w)$

P(s) P(-s)$|_{s = jw}$ = 1 + $\in^2 c_n^{\,2}(1/w)$

For zero location:

$\in^2 c_n^{\,2}(1/w_k)$

$\therefore \in \neq 0 \Rightarrow c_n^{\,2}(1/w_k) = 0$

$c_n(1/w_k) = 0$

Cosn $\cos^{-1}(1/w_k)$ = cos(kπ/2) for k = 1,3,5 ………..(i.e odd)

n$\cos^{-1}(1/w_k)$ = kπ/2

$1/w_k$ = cos(kπ/2n) which gives the zero for inverse chebyshev.

W_k = sec(kπ/2n)

For poles:

1+ $\in^2 c_n^{\,2}(1/w_k) = 0$

The poles location are similar to chebyshev.

Simply replacing w_k by $1/w_k$

i.e if chebyshev poles = p_i

Then , inverse chbyshev poles = $1/p_i$

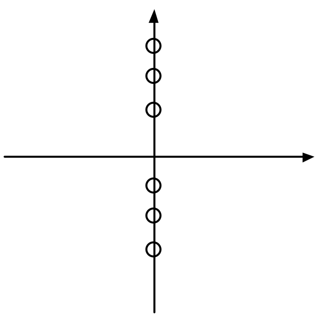


Fig. Pole location

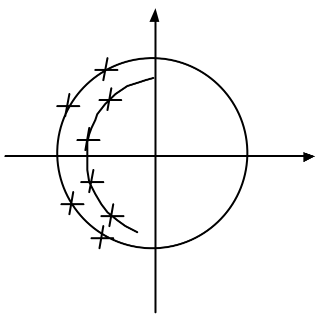


Fig. Zero location

Example:01

Given,

α_{min} = 18 dB

α_{max} = 0.25 dB

w_s = 1.4 rad/sec

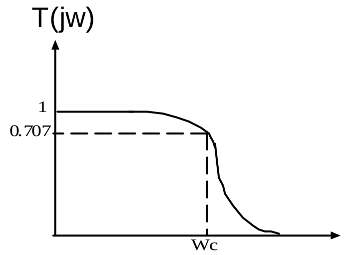
w_p = 1 rad/sec

Find out the pole and zero for inverse chbyshev response.

Chapter 5

Frequency transformation:

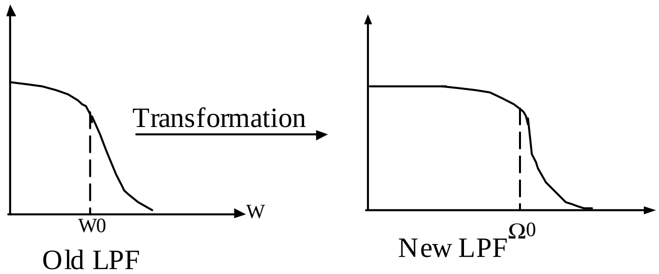
Frequency transformation is important because the prototype LPF with any type of approximation can be converted into high pass band pass , band stops filters within the same characteristics easily.



The effect of frequency transformation are:
1. Magnitude response 2. Network function
3. Location of poles and zeroes. 4. Network elements.

Types of transformation:

1. LP to LP transformation



Replace s by w_o/Ω_o .s
i.e

$\therefore w_0 = 1$ (in normalized case)

$\therefore s \rightarrow s/\Omega_0$

$\therefore T_{LP(new)}(s) = T_{LP(old)}(s/\Omega_0)$

For eamaple,

If

$T_{LP}(s) = 1/S+1$

Then

$T_{LP(old)}(s) = 1/s+1$

$\therefore T_{LP(new)}(s) = T_{LP(old)}(s/\Omega_o) = 1/(s/\Omega_o)+1 = \Omega_0/(s+\Omega_0)$

1. For resistor:
   - No change.
2. For inductor:

$X_L = LS$

Putting $s \rightarrow \frac{s}{\Omega_0}$

$X_L' = L_{old}\ \frac{s}{\Omega_0} = \frac{L_{old}}{\Omega_0} s = L_{new}.S$

$\therefore L_{new} = L_{old}/\Omega_0$

3. For capacitor:

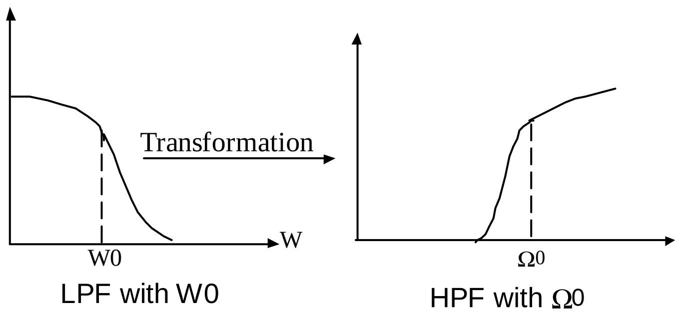
$X_c = 1/cs$

Putting $s \rightarrow \frac{s}{\Omega_0}$

$X_c' = \frac{1}{C_{old}\ \frac{s}{\Omega_0}} = \frac{1}{\frac{C_{old}}{\Omega_0}.s} = \frac{1}{C_{new}.s}$

$\therefore C_{new} = C_{old}/s$

2 LP to HP Transformation:



In this case we replace $s \rightarrow \frac{\Omega_0}{w_0.s}$

Or , $s \rightarrow \frac{\Omega_0}{s}$ [Since $w_0 = 1$]

$\therefore T_{HP}(s) = T_{LP}(s)\Big|_{s = \frac{\Omega_0}{s}} = T_{LP}\left(\frac{\Omega_0}{S}\right)$

Example if $T_{LP}(s) = 1/(s+1)$

Then, $T_{HP}(s) = \frac{1}{\frac{\Omega_0}{s}+1} = \frac{s}{\Omega_0 + s}$

(1) For resistor:

No change

(2) For inductor:

$X_L = LS$

Putting $s \rightarrow \frac{\Omega_0}{s}$

$X_L' = L.\frac{\Omega_0}{s} = \frac{1}{\left(\frac{1}{L\Omega_0}\right).s}$

Comparing $\frac{1}{\left(\frac{1}{L\Omega_0}\right).s}$ with 1/CS

C = $\frac{1}{L\Omega_0}$

(3) For capacitor:
$X_c = 1/cs$

Putting $s \rightarrow \frac{\Omega_0}{s}$

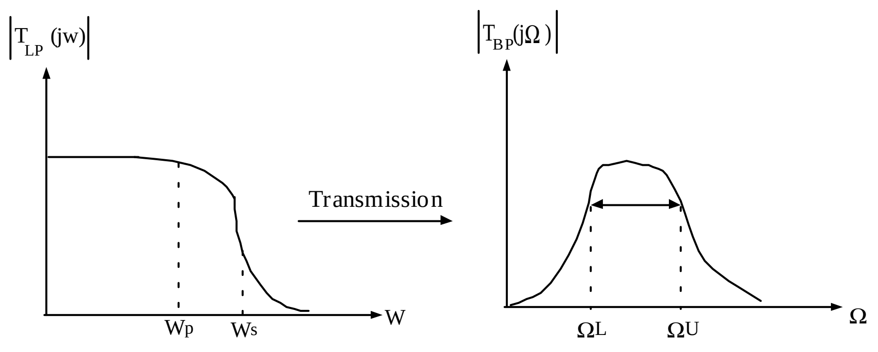
$X_L' = \frac{1}{\left(\frac{\Omega_0}{s}\right).c} = \frac{s}{c\Omega_0} = \left(\frac{1}{C\Omega_0}\right).S = LS$

Comparing $\left(\frac{1}{C\Omega_0}\right).S$ with LS

L = $\left(\frac{1}{C\Omega_0}\right)$

Date: 2056/6/15

(3) LP to BP Transformation:



In this case,

$s \rightarrow w_0.\frac{s^2 + \Omega^2}{\Omega_u - \Omega_L}$

Here, $\Omega_u - \Omega_L = B$
And $w_0 = 1$

$\therefore \; s \rightarrow .\frac{s^2 + \Omega^2}{Bs}$

Where $\Omega_0^2 = \Omega_L . \Omega_u$

(1) For resistor

- no change

(2) For inductor:

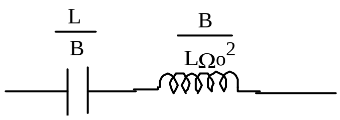
$X_L = LS$

The new value of inductive reactance is given by:

$$X_L' = L.\left(\frac{s^2+\Omega_0^2}{Bs}\right)$$

$$X_L' = \frac{L}{B}.s + \frac{L\Omega_0^2}{Bs} = \frac{L}{B}.s + \frac{1}{\frac{B}{L\Omega_0^2}.s}$$

∴ The new component are inductor and capacitor in series.

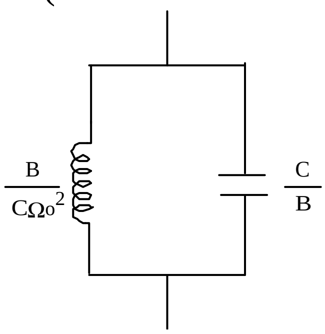


(3) For capacitor:

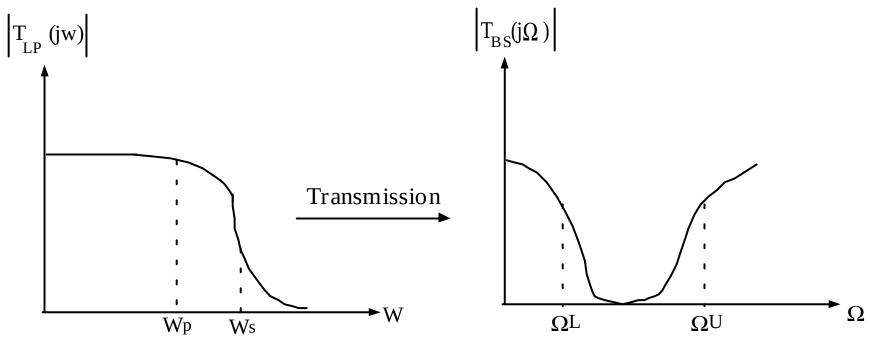
The new capacitive reactance form LP to BP is given by :

$$= \frac{1}{c.\frac{s^2+\Omega_0^2}{Bs}} = \frac{1}{\frac{cs^2+c\Omega_0^2}{Bs}} = \frac{1}{\frac{c}{B}s + \frac{c\Omega_0^2}{Bs}} = \frac{1}{\frac{c}{B}s + \frac{1}{\frac{B}{c\Omega_0^2}.s}}$$

The new components (i.e inductor and capacitor) are in parallel as shown in fig. below:



LP to BS Transformation:



In this case s is replaced by $\frac{Bs}{s^2+\Omega_0^{\ 2}}.w_0$

But $w_0 = 1$, $\therefore\ s \to \frac{Bs}{s^2+\Omega_0^{\ 2}}$

(1) For resistor :

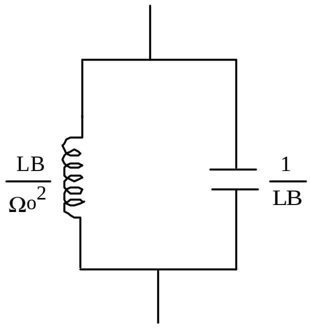
Resistor value remain same.

(2) For inductor:

$X_L = LS$

$X_L = L.\frac{Bs}{s^2+\Omega_0^{\ 2}} = .\frac{1}{\frac{s^2+\Omega_0^{\ 2}}{LBS}} = \frac{1}{\frac{s^2}{LBS}+\frac{\Omega_0^{\ 2}}{LBs}} = \frac{1}{\frac{1}{LB}s+\frac{1}{\frac{LB}{\Omega_0^{\ 2}}.s}}$

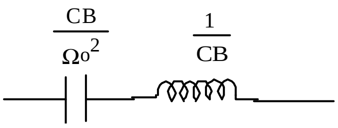
The new component (i.e inductor and capacitor) are in parallel as in figure below:



(3) For capacitor:

$X_c = 1/cs$

$X_c = \frac{1}{c.\frac{Bs}{s^2+\Omega_0^{\ 2}}} = \frac{s^2+\Omega_0^{\ 2}}{CBS} = \frac{s^2}{CBs}+\frac{\Omega_0^{\ 2}}{CBs} = \frac{1}{CB}.s+\frac{1}{\frac{CB}{\Omega_0^{\ 2}}.s}$



Example:01: If $T(s) = \frac{1}{s+1}$, then change the above function from LP to BP. Given , $\Omega_L = 10$ and $\Omega u = 20$.

Solution:

Then, $T_{LP}(s) = \frac{1}{s+1}$, $\Omega_L = 10$, $\Omega u = 20$

We know ,

$\Omega_0^{\ 2} = \Omega_L.\ \Omega_u = 10.\ 20 = 200$

For L_p to BP we replace

$s \to \frac{s^2+\Omega_0^{\ 2}}{B} = \frac{s^2+2000}{(20-10)s} = \frac{s^2+200}{10s}$

Thus,

$$T_{LP}\Big|_{s=\frac{s^2+200}{10s}} = T_{BP}(s) = \frac{1}{\frac{s^2+200}{10s}+1} = \frac{10s}{s^2+10s+200}$$

$$\therefore T_{BP}(s) = \frac{10(s)}{s^2+10s+200}$$

Example:02: Obtain the transfer function of the 4th order Butter worth HPF with $\Omega_0 = 2\pi \times 10^4$ rad/sec.

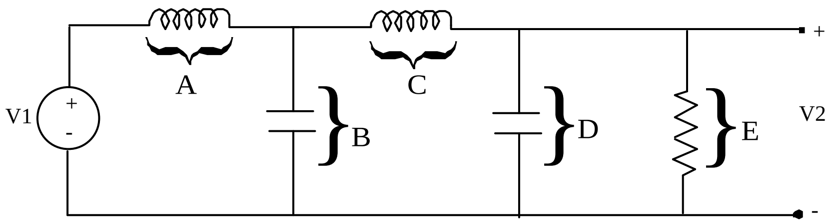
$$T_{LP}(s) = \frac{1}{s^4+2.61313s^3+3.41921s^2+2.61313s+1}$$

We know that ,

$$s \rightarrow \frac{\Omega_0}{s}$$

$$= \frac{1}{\left(\frac{\Omega_0}{s}\right)^4 + 2.61313\left(\frac{\Omega_0}{s}\right)^3 + 3.41921\left(\frac{\Omega_0}{s}\right)^2 + 2.61313\left(\frac{\Omega_0}{s}\right) + 1}$$

Example:03:The filter shown in the figure below is a 4th order chebyshev low pass filter with α_p = 1 dB and w_p = 1. Obtain a bandpass filter from this low pass with Ω_o = 400 rad/sec and B = 150.



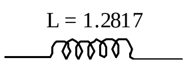
Solution:

For LP to BP conversion , we replace

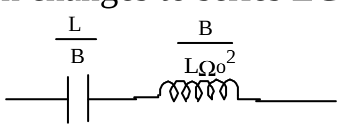
$$s \rightarrow \frac{s^2+\Omega_0{}^2}{Bs}$$

Where, Ω_o = 400 rad/sec , B = 150

Now for section A:



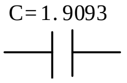
Which changes to series LC component as shown below:



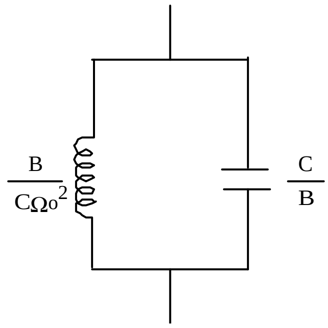
$\therefore$ The new inductor value is = L/B = 1.2817/150 = 8.54 mH

and the new value of capacitor is = B/L Ω^2_0 = 150/(1.2817×400^2) = 731.45 μF.

For section B:

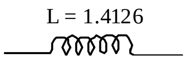


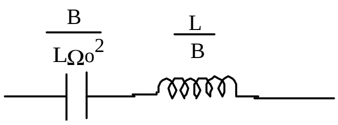
Which changes for LP to BP As:



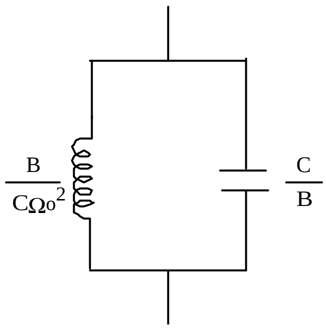
New inductor value = $B/C\,\Omega_o^2$ = $150/(1.9093\times400^2)$= 491.01 μF
New capacitor value = C/B = 1.9093/150 = 12.72 μF

For section C:

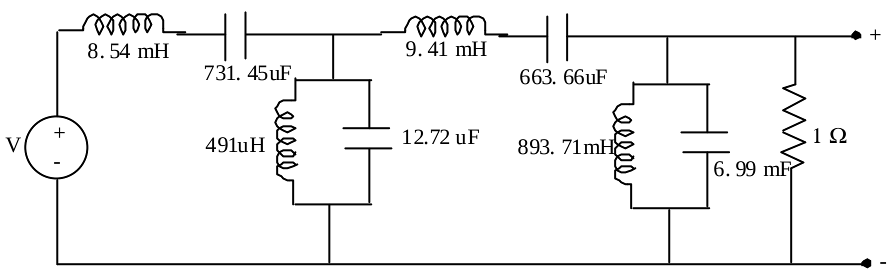




For section D:



For section E:



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Doubly Terminated LC-Ladder ckt:

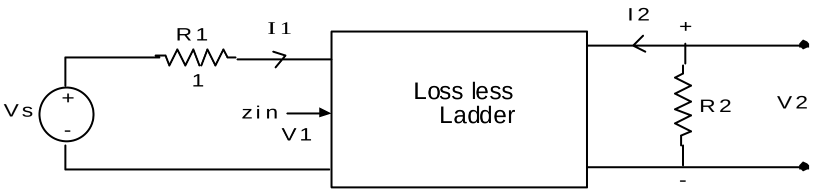


Fig.1 Doubly Terminated LC ladder ckt.

From figure(i)

$I_1 = V_s/(R_1 + V_{in})$ ……………(i)

Where,

$Z_{in} = R_{in} + jx_{in}$ …………….(ii)

Since the ckt is loss less

Input power = output power

$P_1 = z_{in} |I_1(jw)|^2 = |V_2(jw)|^2/R_2$ ……………(iii)

From equation (i) and (iii)

$z_{in}|V_s(jw)|^2/(R_1+z_{in}) = |V_2(jw)|^2/R_2$

or , $|V_2(jw)|^2/|v_s(jw)|^2 = z_{in} R_2/(R_1+z_{in})^2$ …………..(iv)

Now for matched source.

$R_1 = z_{in}$

Which means

$V_1 = v_s/2$

$\therefore P_{1max} = |v_1(jw)|^2/R_1 = |v_s(jw)|^2/4R_1$

Also it is to remember that ,

$P_2 = |v_2(jw)|^2/R_2$

$|(jw)|^2 = p_2/p_{1max} = [|v_2(jw)|^2/R_2]/ |v_s(jw)|^2/4R_1 = 4R_1/R_2. |v_2(jw)/v_s(jw)|^2$ …………..(vi)

Form equation (iv) and (vi)

$|H(jw)|^2 = 4R_1/R_2. \{z_{in} R_2/(R_1+z_{in})\}$

$= 4R_1z_{in}/ (R_1+z_{in})^2 = 1- (R_1-z_{in})^2/(R_1 +z_{in})^2$

$(R_1-z_{in})^2/(R_1 +z_{in})^2 = |(jw)|^2$

= reflection coefficient

$$\rho(s).\rho(-s) = \frac{(R_1 - z_{in})^2}{(R_1 + z_{in})^2}$$

$$\rho(s) = \pm\frac{(R_1 - z_{in})}{(R_1 + z_{in})} \quad \text{....................(vii)}$$

From equation (vii) , we get

$$z_{in} = R_1.\frac{1-\rho(s)}{1+\rho(s)} \quad \text{——— 1st } z_{in}$$

$$z_{in} = R_1.\frac{1+\rho(s)}{1-\rho(s)} \quad \text{----------- 2nd } z_{in}$$

……..(viii)

Generally we take $R_1 = 1$. Both impedances in equation (viii) are reciprocal impedance.

Synthesis of Doubley Terminated LC ladder with equal terminal (All pass filter)

For butterworth response:

$$|T(jw)|^2 = |H(jw)|^2 = \frac{1}{1+w^{2n}} = \frac{N(s)N(-s)}{D(s)D(-s)} \qquad \text{[since } w_0 = 1\text{]}$$

$$|\rho(s)|^2 = 1 - |H(s)|^2 = 1 - |H(jw)|^2 = 1 - \frac{1}{1+w^{2n}} = \frac{w^{2n}}{1+w^{2n}}$$

$$\rho(s).\rho(-s) = \frac{w^{2n}}{1+w^{2n}} = \frac{w^{2n}}{D(s).D(-s)} = \frac{s^n.(-s)^n}{D(s).D(-s)} \quad \ldots\ldots\text{(ix)}$$

Now,

For n = 1

D(s) = s+1 [since T(s) = H(s) = 1/S+1]

Form equation (ix)

ρ(s) = s^n/D(s)

= s^1/s+1 = s/s+1

$$\therefore z_{in1} = R_1.\frac{1-\rho(s)}{1+\rho(s)} = 1.\frac{1-\frac{s}{s+1}}{1+\frac{s}{s+1}} = .\frac{s+1-s}{s+1+s}$$

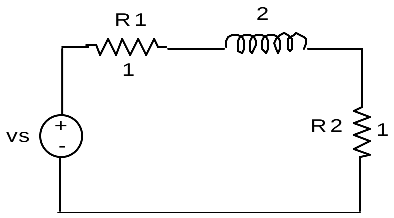
$Z_{in1} = .\frac{1}{2s+1}$ ....(a)

Z_{in2} = 2s+1 ……….(b)

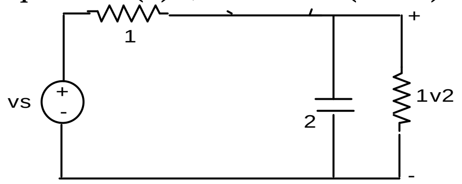
Z_{in2} = 2s+ 1 = Ls + R

i.e L = 2, and R = 1

∴ The ckt will be



From equation (a) , z_{in1} = 1/(2s+1) i.e c = 2, and R = 1



For n = 2

D(s) = $s^2+\sqrt{2}s + 1$

$$\rho(s) = \frac{s^n}{D(s)} = \frac{s^n}{s^2+\sqrt{2}s+1}$$

$$\therefore z_{in1} = \frac{1-\rho(s)}{1+\rho(s)} = \frac{1-s^2/(s^2+\sqrt{2}s+1)}{1+s^2/(s^2+\sqrt{2}s+1)} = \frac{(s^2+\sqrt{2}s+1-s^2)}{(s^2+\sqrt{2}s+1+s^2)}$$

$$\therefore z_{in1} = \frac{(\sqrt{2}s+1)}{(2s^2+\sqrt{2}s+1)} \quad \text{........(a)}$$

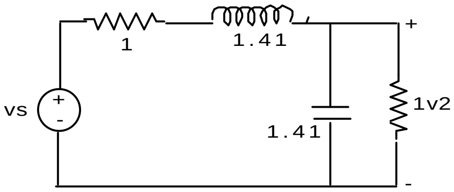
Similary,

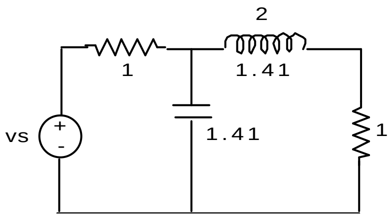
$$Z_{in2} = \frac{2s^2+\sqrt{2}s+1}{\sqrt{2}s+1} \quad \text{..........(b)}$$

Taking equation (b)

√2 s+1) 2s²+√2 s +1 (√2.s $z_1(s)$
2s²+√2 s
1) √2.s + 1(√2 s $Y_2(s)$
√2 s
1) 1 (1 $z_3(s)$
1

∴ The ckt will be as follows:





Home work : For n = 3 and n = 4

Date: 2065/6/17

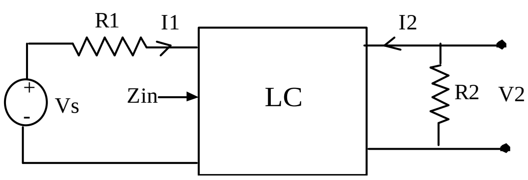
Synthesis of Doubly Terminated LC - Ladder with unequal termination: ($R_1 \neq R_2$) :

For $R_1 \neq R_2$ the butter worth response is given by ,

$$|H(jw)|^2 = \frac{H^2(0)}{1+w^{2n}} = |T(jw)|^2$$

Generally we take,

$R_1 \neq 1$ and $R_1 \neq R_2$



From figure, the transform function , T(s) = $\frac{V_2}{V_s}$

From which we get ,

T(0) = $\frac{R_2}{R_2 + R_1}$

Now we know

$$|H(s)|^2 = \frac{4R_1}{R_2}.\left|\frac{V_2(s)}{V_s(s)}\right|^2$$

$$|H(s)|^2 = \frac{4R_1}{R_2}.|T(s)|^2$$

$$|H(S)| = 2\sqrt{\frac{R_1}{R_2}}.T(s)$$

$$|H(0)| = 2\sqrt{\frac{R_1}{R_2}}.T(0) = 2\sqrt{\frac{R_1}{R_2}}.\frac{R_2}{R_1 + R_2}$$

$$\therefore\ |H(0)| = 2\frac{\sqrt{R_2.R_1}}{\sqrt{R_1 + R_2}}$$

Example:01: Realize the doubly terminated ladder filter with a Butter worth response for n = 3, R_1 = 1, R_2 = 2 .

Solution:

We know, for unequal termination (i.e $R_1 \neq R_2$) the Butterworth response is given by,

$$|H(jw)|^2 = \frac{H^2(0)}{1 + w^{2n}}$$

Here, n = 3, R_1 = 1 & R_2 = 2

$$H^2(0) = \frac{4R_2.R_1}{(R_2 + R_1)^2} = \frac{4.1.2}{(1+2)^2} = \frac{8}{9}$$

$$\therefore\ |H(jw)|^2 = \frac{8/9}{1 + w^{2n}}$$

The reflection coefficient function is

$$|\rho(jw)|^2 = 1 - |H(jw)|^2$$

$$= 1 - \frac{8/9}{1 + w^{2n}} = \frac{1 + w^{2n} - 8/9}{1 + w^{2n}} = \frac{1/9 + w^{2n}}{1 + w^{2n}}$$

$$|\rho(jw)|^2 = \frac{1/9 - (s/j)^{2\times 3}}{1 + w^{2\times 3}} = \frac{1/9 + (s/j)^6}{1 + w^6}$$

Or, $|\rho(s)|^2 = \frac{1/9 - (s)^6}{1 - s^6} = \frac{(1/3)^2 - (s^3)^2}{1 - s^6} = \frac{(1/3 - s)(1/3 + s)}{1 - s^6}$

$$\rho(s).\rho(-s) = \frac{(1/3 + s^3)}{D(s)}.\frac{(1/3 - s^3)}{D(-s)}$$

Where, D(s). D(-s) = 1- s^6

$$\therefore\ \rho(s) = \frac{1/3 + s^3}{D(s)}$$

For n = 3,

D(s) = s^3+2s^2 + 2s+1   (from table)

∴ The first impedance is ,

$$Z_{in1} = \frac{1-\rho(s)}{1-\rho(s)} = \frac{1 - \dfrac{1/3 + s^3}{s^3 + 2s^2 + 2s + 1}}{1 + \dfrac{1/3 + s^3}{s^3 + 2s^2 + 2s + 1}}$$

$$Z_{in1} = \frac{2s^2 + 2s + 2/3}{2s^3 + 2s^2 + 2s + 4/3} \quad \text{……….(a)}$$

$$Z_{in2} = \frac{2s^3 + 2s^2 + 2s + 4/3}{2s^2 + 2s + 2/3} \quad \text{…………..(b)}$$

Now using continued fraction method for equation (b)

$2s^2+2s+2/3$) $2s^3+2s^2+2s+4/3$ (s $z_1(s)$

$2s^3+2s^2+2/3.s$

4/3.s+4/3) $2s^2+2s+2/3$ (3/2. s $Y_2(s)$

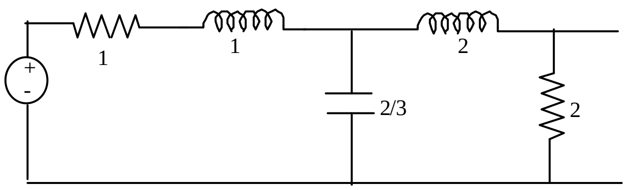
$2s^2+2s$

2/3) 4/3.s +4/3(2s $z_3(s)$

4/3.s

4/3) 2/3 (1/2 $Y_4(s)$

2/3



Home Assignment:

- Try it for n = 1, 2, 3 and 4 , for unequal terminal i. e R_1 = 1 and R_2 = 2.

[for n = 4, D(s) = s^4+2.16s^3+3.14s^2+2.6s+1]

- Review of ideal and non ideal properties of operational amplifiers, GBP, CMRR, Inverting and non inverting A/F.

Fundamental of Active filter circuit:-

- Ideal & Non-ideal properties of op-amp.
- Gain Bandwidth product(GBP)
- CMRR & its importance.

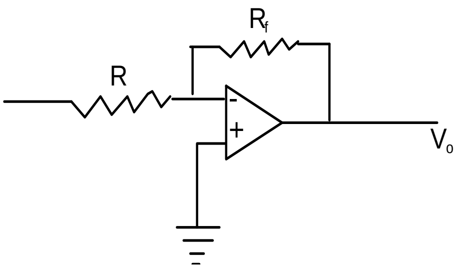
The main advantage of Active filter:-

- Small in size
- Provide grater amplification
- Cheaper than passive filter.

The limitation area:-

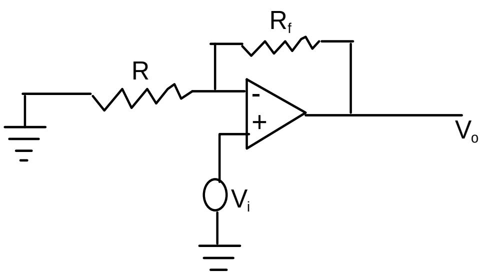
- Extra V_{cc} is required
- Sensitive to temperature
- Low gain at high temperature
- Low gain at high frequencies
- CMRR should be high

Certain important configuration of op-amp:-



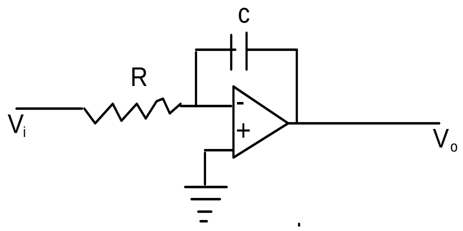
$$V_o = -\frac{R_f}{R}.Vi$$

(2) Non-investing:-



$$Vo = \left(1 + \frac{RF}{R}\right)Vi$$

(3)Integration:-

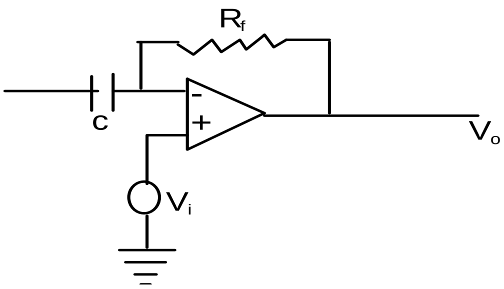


$$Vo = -\frac{1}{RCS}.Vi = \frac{1}{RC}\left(-\frac{1}{S}\right)Vi$$

If R1=1 & C = 1, then

$\frac{Vo}{Vi} = -\frac{1}{S}$ I.e. Integrator always contributes polo.

(4) **Differentiator:-**



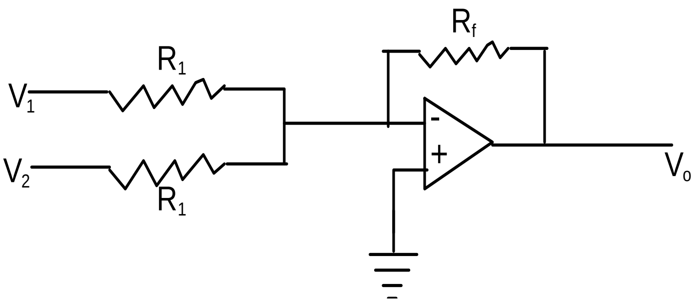
$$\frac{Vi - O}{\frac{1}{CS}} = \frac{Vo - O}{R}$$

$$Vo = -(CRS)Vi$$

If Ro= 1& Co= 1, Then

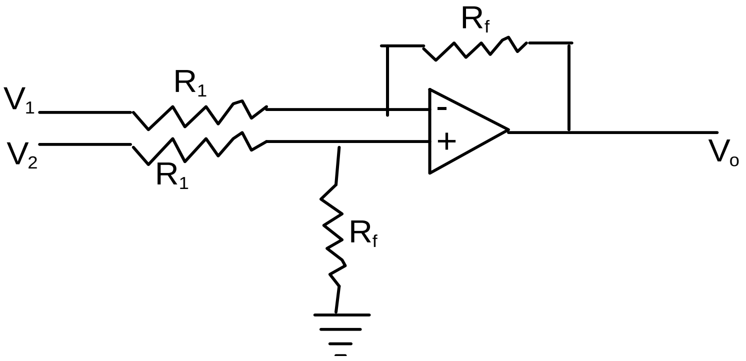
$$\frac{Vo}{Vi} = -S$$

(5) **Summer:-**



$$Vo = -\frac{RF}{Ri}(V_1 + V_2)$$

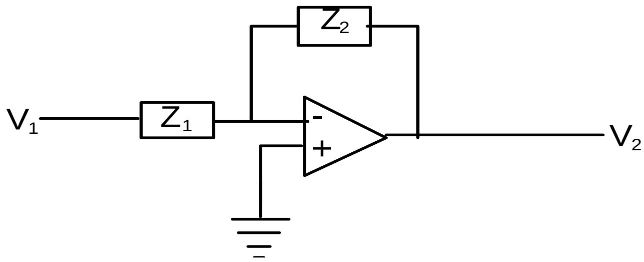
(6) **Subtract or (Difference A/F)**



$$Vo = \frac{RF}{Ri}(V_2 - V_1)$$

Design of Active filters (op-amp based):-

(1) Investing type:-



From fig.

$$R(S) = \frac{V_1(S)}{V_1(S)} = \frac{-Z_2}{Z_1}$$

(a) T(S) = -K/S

Since, the above T(S) contributes polo we can reduce the T(S) with T(S) of integrator

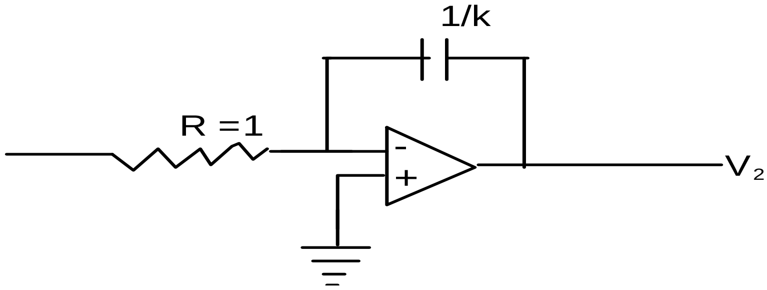
I.e. $T(S) = -\frac{1}{RCS} = \frac{-K}{S}$

$$\Rightarrow K = \frac{1}{RC}$$

If R=1, then,

C=1/K

$\Rightarrow$ If C=1, then, R=1/K



Thus the design will be

(b) T(S)= -KS (Do yourself)

(c) T(S) = -K(S+a_1)

We can compare with the general T(S) of investing type ie.

$$T(S) = -\frac{Z_2}{Z_1}$$

$$\therefore \frac{Z_2}{Z_1} = k(S + a_1)$$

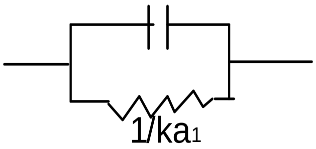
$$\text{Or } \frac{\frac{1}{y_2}}{\frac{1}{y_1}} = \frac{y_1}{y_2} = k(S + a_1)$$

If $y_2 = 1$, then,

$Y_1 = KS + Ka_1$

$\therefore Y_2$

Y_1



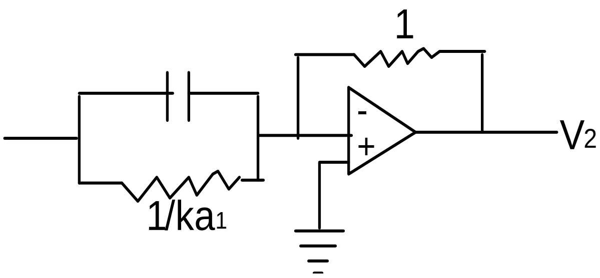


Fig:- Design for R(s) = - (s + a_1)

(d) $T(S) = -\frac{K}{S + P_1}$

Let we can write,

$$\frac{Z_2}{Z_1} = \frac{K}{S + P_1}$$

$$\frac{y_1}{y_2} = \frac{K}{(S + P_1)}$$

$$\frac{y_1}{y_2} = \frac{1}{\frac{(S + P_1)}{K}}$$

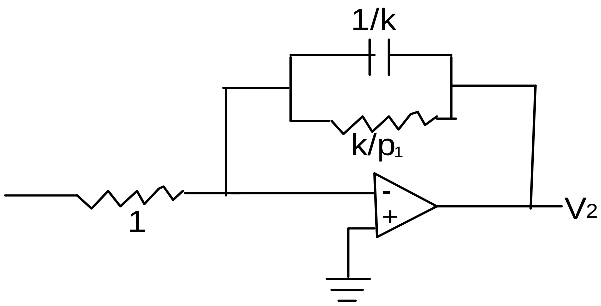


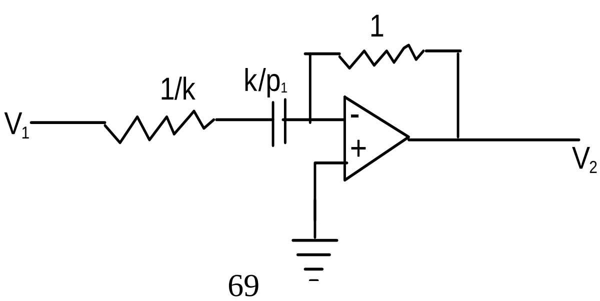
fig: Design for T(S)= -K/(S+P_1)

y_1=1, then

$$y_2 = \frac{S + P_1}{K} = \frac{S}{K} + \frac{P_1}{K}$$

(e) $T(S) = \frac{-ks}{s + p_1}$

$$\frac{Z_2}{Z_1} = \frac{1}{\frac{1}{K} + \frac{P_1}{KS}}$$



If Z_2=1, then,

$$Z_1 = \frac{1}{K} + \frac{P_1}{KS}$$

(f) $T(S) = -K\frac{S+q_1}{S+P_1}$

$$\frac{Z_2}{Z_1} = K\left(\frac{s+q_1}{s+p_1}\right)$$

$$\frac{y_1}{y_2} = k\left(\frac{s+q_1}{s+p_1}\right)$$

Let $y_1 = ks + kq_1$

Then, $y_2 = s + p_1$

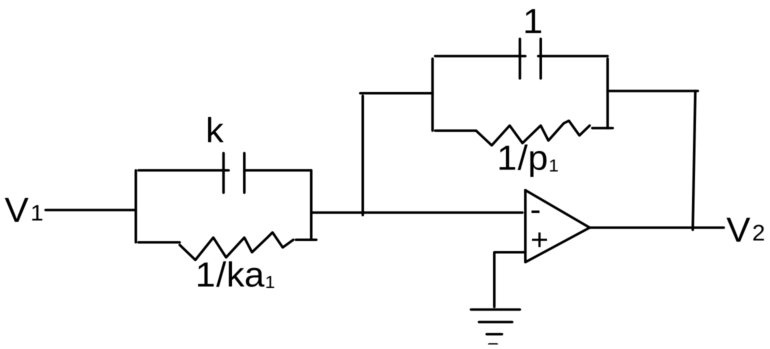
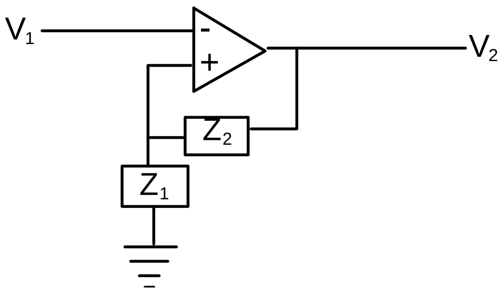


Fig: Design for T(S) = $\frac{-k(s+q_1)}{(s+p_1)}$

2nd approach of above problem (म्य थ्यगचकभाी)

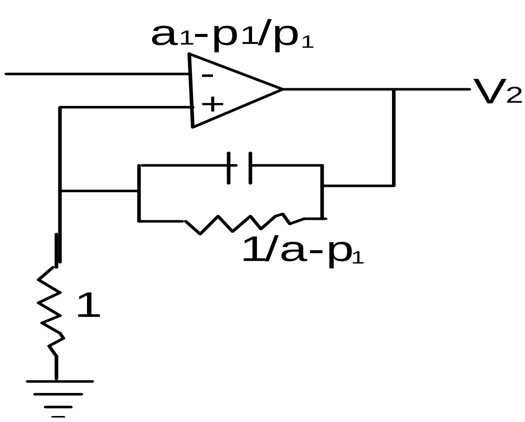
(2)**Non-investing type:-**



(a) T(S) = $\frac{k(s+q_1)}{(s+p_1)}$ Where, $q_1 > p_1$

Comparing,

$$1 + \frac{z_2}{z_1} = k\left(\frac{s+q_1}{s+p_1}\right)$$



$$\frac{z_2}{z_1} = k\left(\frac{s+q_1}{s+p_1}\right) - 1$$

$$= \frac{ks + kq_1 - s - p_1}{s + p_1}$$

$$\frac{z_2}{z_1} = \frac{s(k-1) + (kq_1 - p_1)}{(s + p_1)}$$

For, k = 1

$$\frac{z_2}{z_1} = \frac{q_1 - p_1}{s + p_1} \qquad T(s) = \frac{k(s+q_1)}{(s+p_1)} \text{ for } k = 1$$

$$\frac{y_1}{y_2} = \frac{1}{\frac{s}{q_1 - p_1} + \frac{p_1}{q_1 - p_1}}$$

If $y_1 = 1$, then

$$y_2 = \frac{s}{q_1 - p_1} + \frac{p1}{q_1 - p_1}$$

For, $k \neq 1$

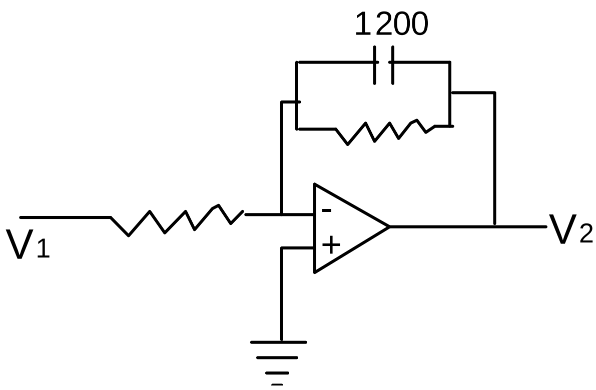
$$\frac{z_2}{z_1} = \frac{s(k-1) + (kq_1 - p_1)}{(s + p_1)}$$

We assume,

$Kq_1 = p_1$

$K = p_1/q_1$

$$\therefore \frac{z_2}{z_1} = \frac{s(k-1)}{(s+p_1)} = \frac{\frac{s(p_1 - q_1)}{q_1}}{(s+p_1)}$$



$$\frac{z2}{z1} = \frac{\frac{p_1 - q_1}{q_1}}{1 + p_1/s}$$

If $z_2 = \frac{p_1 - q_1}{q_1}$ Design for $T(s) = \frac{k(s+q_1)}{(s+p_1)}$

Then, $z_1 = 1 + p_1/s$ for $k \neq 1$ & $p_1 > q_1$

2nd approach $\frac{y_1}{y_2}$ =? (**Do Yourself**)

Example:- 01

Realize 1st order inverting which satisfy the following T(s)

$$T(s) = \frac{1000}{s + 1000}$$

We know that,

$$\frac{-z_2}{z_1} = \frac{-1000}{s + 100}$$

$$\frac{y_1}{y_2} = \frac{1000}{s+1000} = \frac{1}{\frac{s}{100}+1}$$

If y_1=1 then,

$$y_2 = \frac{s}{1000}+1,\ c = \frac{1}{1000}$$

Example:-02

Realize the 1st order Non- inverting filter with following T(s)

$$T(s) = \frac{s+4}{s+8}$$

soln:- For the case given,

$$T(S) = 1+\frac{z_2}{z_1} = \frac{s+4}{s+8}$$

$$\text{Or, } \frac{z_2}{z_1} = \frac{s+4-s-8}{s+4} = \frac{-4}{s}$$

$\therefore$ The direct approach does not provide the required design; we go in the following manner.

Here, P_1 =8

Q_1=4

Ie, p_1=q_1

$$\text{Let, } k = \frac{p_1}{q_1} = \frac{8}{4} = 2$$

$$\therefore T(s) = \frac{2(s+4)}{(s+8)}.\frac{1}{2} \qquad = \left[2\frac{(s+4)}{(s+8)}\right]\left[-\frac{1}{2}\right](-1)$$

$$= T_1(s).T_2(s).T_3(s)$$

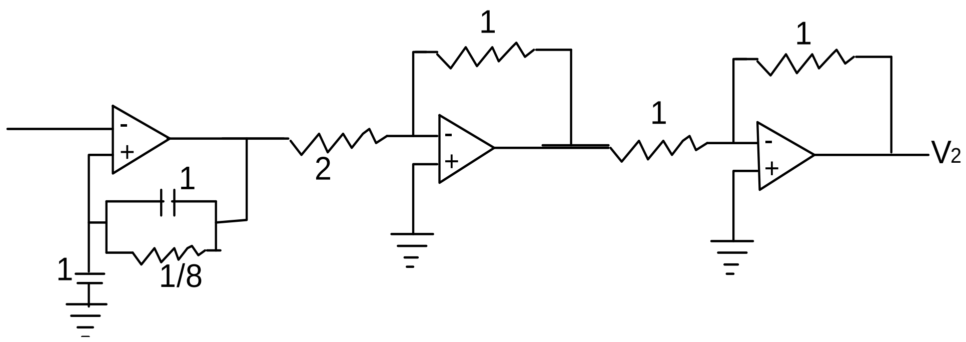
$$\text{For, } T_1(S) = \frac{2(s+4)}{(s+8)}$$

$$1+\frac{z_2}{z_1} = \frac{2s+8-s-8}{(s+8)}$$

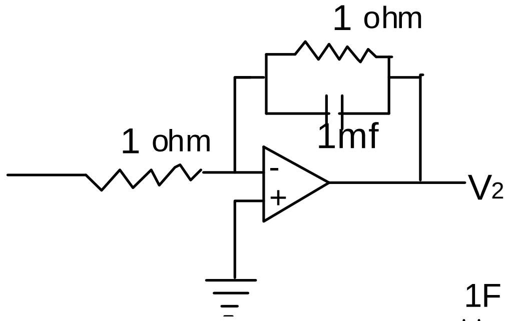
$$\frac{z_2}{z_1} = \frac{s}{s+8}$$

$$\frac{y_1}{y_2} = \frac{s}{s+8}$$

If y_1= S, then y_2 = S + 8



RC-CR Transformation:-

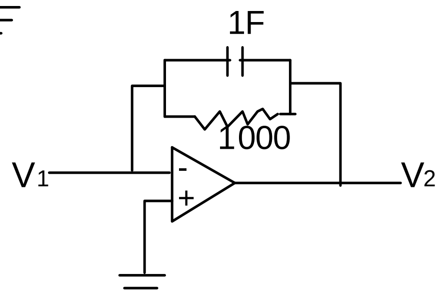


$$T(S) = \frac{z_2}{z_1} = \frac{1000}{S + 1000}$$

It is law pass response,

If, $Ri = \frac{1}{Ci} = \frac{1}{1mf} = 1k\Omega$

$$\& \ Ci = \frac{1}{Ri} = \frac{1}{1} = 1F$$



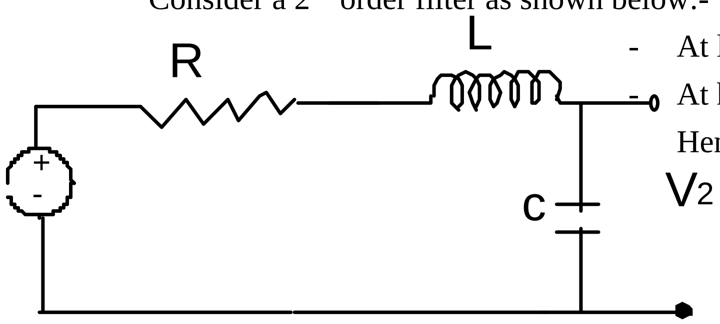
$$\text{T(s)} = \frac{z_2}{z_1} = \frac{s}{s + 1000}$$

It is the transformation by which a low pass filter can be converted into a high pass filter by the simple change in the component Value i.e. In this case R_i is replace by C_i and C_i=1/R_i and C_i is replaced by R_i and R_i=1/C_i.

CHAPTER:- 8

Biquad circuits:-

Consider a 2nd order filter as shown below:-



- At low freq, c behaves as line open cut so, $V_2 = V_1$
- At high freq. c behave as line short cut V_2= O

Hence, it is a low pass filter.

$$\text{T(s)} = \frac{v_2(s)}{v_1(s)} = \frac{\frac{1}{cs}}{R + LS + \frac{1}{cs}}$$

$$= \frac{\frac{1}{Lc}}{s^2 + \frac{R}{L}s + \frac{1}{Lc}}$$

To get poles, $S2+\frac{R}{L}S+\frac{1}{LC}=0$

&, for loss less ckt , ie, if R= 0,

Then, $S^2 + 1/Lc = 0$

Or, $s=\pm j\frac{1}{\sqrt{LC}}=\pm jwo$ where, $wo=\frac{1}{\sqrt{Lc}}$

$\therefore$ Poles are imaginary and conjugate,

Quality factor:-(Q)

$$Q=\frac{WoL}{R}$$

It is defined as the ratio of inductive reactance at frequency W_o to the resistance.

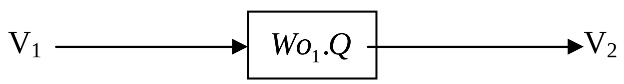
Now,

$$Q=\frac{1}{\sqrt{LC}}.\frac{L}{R}=\frac{1}{R}\sqrt{\frac{L}{C}}$$

Also, $\frac{Wo}{Q}=\frac{R}{L}$

$$\therefore T(s)=\frac{Wo^2}{S^2+\frac{Wo}{Q}S+Wo^2} \qquad \text{..............................} \quad (i)$$

This is the standard from & the design parameter is Wo & Q.



To get the actual poles:- $S^2+\frac{Wo}{Q}.S+Wo^2=0$

Let, the poles be, $-\varepsilon \pm j\beta$ then,

D(S) = $(S+\alpha+j\beta)(s+\alpha-j\beta)$

D(S) = $S^2+2\alpha\beta+(\alpha^2+\beta^2)=0$ .... (ii)

Comparing equation (i) and (ii)

$2\alpha=\frac{Wo}{Q}$ & $\alpha^2+\beta^2=Wo^2$

$$\therefore \alpha=\frac{Wo}{2Q}$$

$$\& \ \beta=Wo\sqrt{1-\frac{1}{4Q^2}}$$

Typing Biquad current:-

A typical Biquad ckt can be represented as, T(s) = $\pm\frac{Gw_0^2}{s^2+\frac{w_0}{Q}s+w_0^2}$

Where, G = Gain & $\pm$ choice of inverting and non inverting.
In normalized case, i.e. for $w_0 = 1$

$$T(s)=\frac{\pm G}{s^2+\frac{s}{Q}+1} \text{ ……….. (i)}$$

Equation (i) can be implemented if G & Q ate given,
Let us go for inverting type of design

i. e. $T(s)=\frac{-G}{s^2+\frac{s}{Q}+1}$

$$\frac{v_2}{v_1}=-\frac{G}{s^2+\frac{s}{Q}+1}$$

$$-Gv_1=\left(s2+\frac{s}{Q}+1\right)v_2$$

$$-Gv_1=\left[s\left(s^2+\frac{1}{Q}\right)+1\right]v_2$$

$$v_2=-\frac{Gv_1}{s\left(s+\frac{1}{Q}\right)}-\frac{v_2}{s\left(s+\frac{1}{Q}\right)}$$

$$v_2=-\frac{v_2}{\left(s+\frac{1}{Q}\right)}-\frac{v_2}{s\left(s+\frac{1}{Q}\right)}$$

$$v_2=\left[\frac{-vs}{\left(s+\frac{1}{Q}\right)}-\frac{Gv_1}{\left(s+\frac{1}{Q}\right)}\right]\left[-\frac{1}{s}\right][-1] \text{………. (ii)}$$

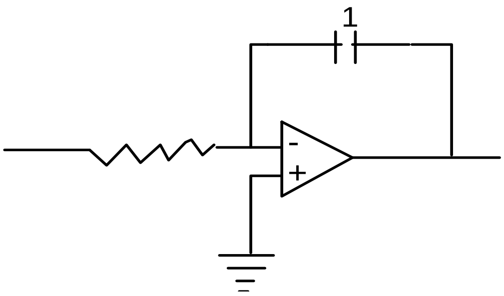
The equation (ii) is cascade Realization costing of 3 steps:

Stage:-1

$$-\frac{1}{s+\frac{1}{Q}}v_2+\left(\frac{-G}{\left(s+\frac{1}{Q}\right)}\right)v_1 \text{………… ii (a)}$$

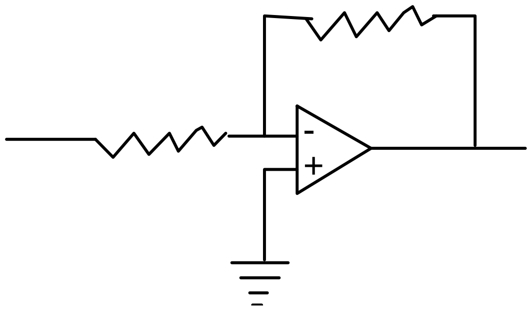
Stage:-2

$$-\frac{1}{s} \text{ ……….. ii (b)}$$

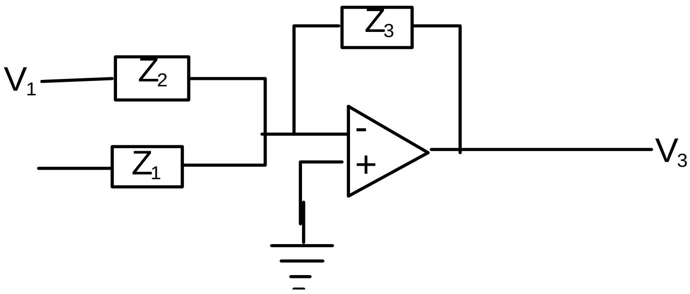


Stage:-3

(-1) = ii(c)



For stage 1, we need more analysis:



From figure

$$v_3 = -\frac{z_3}{z_1}.v_1 - \frac{z_3}{z_2}.v_2$$

$$v_3 = z_3\left[\left(\frac{-1}{z_2}\right)v_2 + \left(\frac{-1}{z_1}\right)v_1\right] \text{………..} \text{ (iii)}$$

From equation ii (a)

$$v_3 = \frac{1}{s+\frac{1}{Q}}\left[-1v_2 + (-G)\right]v_1$$

$$\therefore v_3 = \frac{1}{s+\frac{1}{Q}}\left[\frac{-v_2}{1} + \left(-\frac{1}{\frac{1}{G}}\right)\right]v_1 \text{…………. (iV)}$$

Comparing eq^n (iii) & (iV)

$$z_3 = \frac{1}{s+\frac{1}{Q}}$$

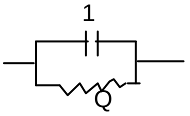
$z_2 = 1$ (a resistor)

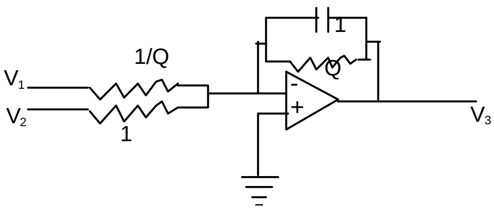
$z_2 = -\frac{1}{G}$ (a resistor)

$$For, z_3 = \frac{1}{s + \frac{1}{Q}}$$

Or, $y_3 = s + \frac{1}{Q}$

$\therefore$ The ckt for Z_3 will be





The overall ckt will be,

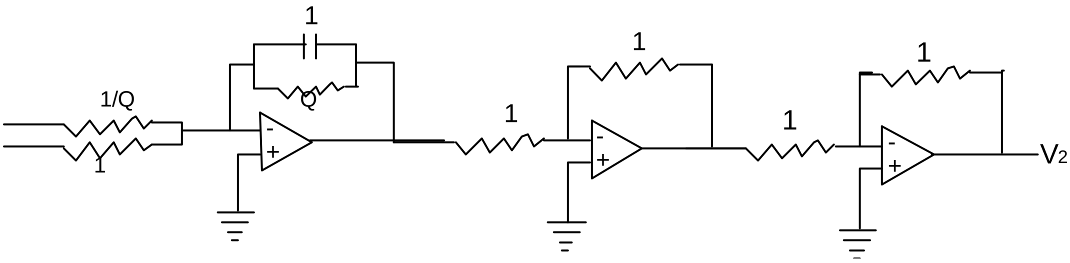


Fig: This is ring of 3 ckt and is popularly known as two Thomas Biquid.

Two Thomas Biquid:-

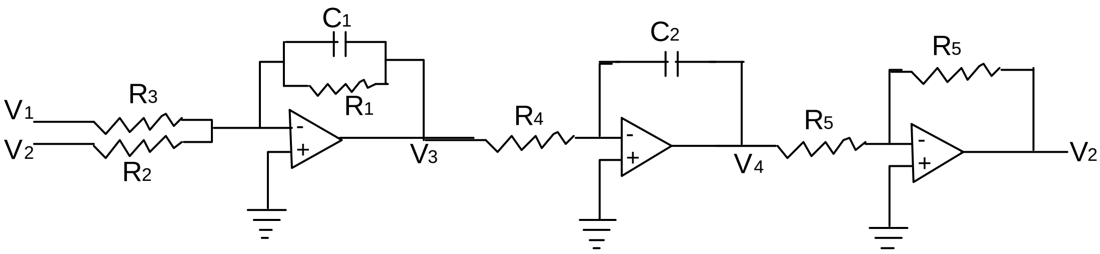


Fig:-General Two Thomas Biquid.

From figure,

$$v_3 = \frac{-R_1 \frac{1}{c_1 s}}{R_1 + \frac{1}{c_1 s}} \left[\frac{v_2}{R_2} + \frac{v_1}{R_3} \right] \text{……… (i)}$$

$$v_4 = -\frac{1}{R_4 c_2 s} v_3 \text{……… (ii)}$$

$$v_2 = -v_4 \text{……… (iii)}$$

From eq^n (ii) & (iii)

$$v_2 = -\frac{1}{R_4 c_2 s} v_3 \text{……… (iv)}$$

Again, from eq^n (i) & (iv)

$$R_4c_2s.v_2 = \frac{-R_1}{\frac{c_1s}{R_1+\frac{1}{c_1s}}\left[\frac{v_2}{R_2}+\frac{v_1}{R_3}\right]}$$

$$\therefore T(s) = \frac{v_2}{v_1} = \frac{-\frac{1}{R_3R_4c_1c_2}}{s^2+\frac{1}{Rc_1}s+\frac{1}{R_2R_4c_1c_2}} \text{ ………… (V)}$$

But the standard form of Biquid is

$$T(s) = \frac{v_2}{v_1} = \frac{-Gw_0^{\;2}}{s^2+\frac{w_0}{Q}.s+w_0^{\;2}} \text{ ……… (Vi)}$$

Comparing eqn (V) & (Vi)

$$w_0^{\;2} = \frac{1}{R_2R_4c_1c_2}$$

$$\therefore w_0^{\;2} = \frac{1}{\sqrt{R_2R_4c_1c_2}} \text{ ……… (Vii)}$$

Also, $\frac{1}{R_2R_4c_1c_2} = Gw_0^{\;2}$

or, $\frac{1}{R_2R_4c_1c_2} = G.\frac{1}{R_2R_4c_1c_2}$

or, $G = \frac{R_2}{R_3}$ …….(Viii)

Finally,

$$\frac{w_0}{Q} = \frac{1}{R_1c_1}$$

or, $\frac{1}{\sqrt{\frac{R_2R_4c_1c_2}{Q}}} = \frac{1}{R_1c_1}$

$$\Rightarrow Q = \sqrt{\frac{R_1^{\;2}c_1}{R_2R_4c_1c_2}} \text{ ……… (ix)}$$

With,

$c_1 = c_2 = 1$

& $R_2 = R_4 = 1$

We get,

$W_0 = 1$

$G = \frac{1}{R_3}$

$R_3 = \frac{1}{G}$

$Q = R_1$

$\Rightarrow R_1 = Q$

The important property of the Biquid ckt is that it can be orthogonally turned. It means

(a) R_2 can be adjusted to a specified Value of $w_{0.}$

(b)R_1can then be adjusted to give specified of Q without changing w_0, which has been already adjusted.

(3) Finally, R_3can be adjusted to give the desired Value of G fir the ckt without changing w0 & Q which has already been set.

These three steps are known as tuning algorithm.

Sallen – key Biquad circuit:-

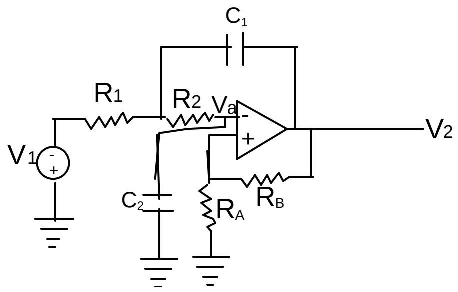


Fig: Sallen-key Biquad

From fig (i),

$$\frac{v_2}{v_1} = 1 + \frac{R_A}{R_B} = k \quad \ldots\ldots\ldots\ldots\ldots\ldots .(i)$$

$$\Rightarrow Va = \frac{v_2}{k} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (ii)$$

Applying Nodel Analysis at mode a,

$$\frac{V_a - V_b}{R_2} + \frac{V_a - 0}{\frac{1}{C_2}S} = 0 \quad \ldots\ldots\ldots\ldots\ldots (iii)$$

Applying Nodal analysis at node b,

$$\frac{v_b - v_1}{R_1} + \frac{v_b - \frac{v_2}{k}}{R_2} + \frac{v_b - v_2}{\frac{1}{c_1 s}} = 0$$

$$\text{Or,} V_b\left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right) - \frac{v_2}{R_2 k} - \frac{v_1}{R_1} - v_2 c_1 s = 0$$

$$\text{Or,} V_b\left(\frac{1}{R_1} + \frac{1}{R_2} + c_1 s\right) - \frac{v_1}{R_1} - v_2\left(\frac{1}{R_2 k} + c_1 s\right) = 0$$

Similarly, rearranging eq^n (iii)

$$\frac{\frac{v_2}{k}}{R_2} - \frac{v_3}{R_2} + \frac{\frac{v_2}{k}}{\frac{1}{c_2 s}} = 0$$

$$\left(\frac{1}{R_2 k} + \frac{c_2 s}{k}\right) v_2 - \frac{v_b}{R_2} = 0$$

Or, $v_b = R_2\left(\frac{1}{R_2 k} + \frac{c_2 s}{k}\right)v_2$ ……….(Vi)

Thus from eq^n (V) & (Vi)

$$R_2\left(\frac{1}{R_2 k} + \frac{c_2 s}{k}\right)\left(\frac{1}{R_1} + \frac{1}{R_2} + c_1 s\right)v_2 - \frac{v_1}{R_1} - v_2\left(\frac{1}{R_2 k} + c_1 s\right) = 0$$

Or, $\left[R_2\left(\frac{1}{R_2 k} + \frac{c_2 s}{k}\right)\left(\frac{1}{R_1} + \frac{1}{R_2} + c_1 s\right) - \left(\frac{1}{R_2 k} + c_1 s\right)\right]v_2 = \frac{v_1}{R_2}$

$$\Rightarrow T(s) = \frac{v_2}{v_1} = \frac{k\frac{1}{R_1 R_2 c_1 c_2}}{s^2 + \left(\frac{1}{R_1 c_1} + \frac{1}{R_2 c_1} + \frac{(1-k)}{R_2 c_2}\right)s + \frac{1}{R_1 R_2 c_1 c_2}}$$

$$T(s) = \frac{G w_0^{\;2}}{s^2 + \left(\frac{w_0}{Q}\right)s + w_0^{\;2}}$$ ………. (Viii)

Comparing eq^n (Vii) and (Viii)

G = k

$$w_0 = \frac{1}{\sqrt{R_1 R_2 c_1 c_2}}$$

$$\frac{w_0}{Q} = \frac{1}{R_1 c_1} + \frac{1}{R_2 c_1} + \frac{1-k}{R_2 c_2}$$

<u>Design – I (equal elements Values):-</u>

In this case,

$R_2 = R_1 = R = 1$

& $C_1 = C_2 = C = 1$

For which,

$W_0 = 1$

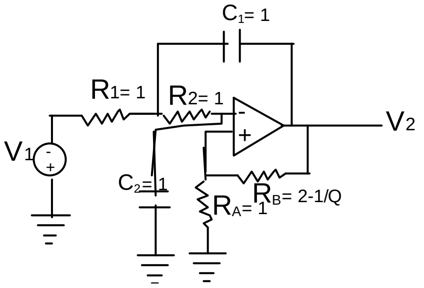
K = $3 - \frac{1}{Q} = 1 + \frac{R_B}{R_A}$

Now, let us take,

$R_A = 1$ then,

$R_B = 2 - 1/Q$

$\therefore$ In this case the final ckt will be,



Design –II (unity gain design):-

In this case k = 1

This is turn require d the non – inverting ckt to be replaced by a Voltage followers.

We keep,

$R_1 = R_2 = 1$

We know,

$$\frac{w_0}{Q} = \frac{1}{R_1 c_1} + \frac{1}{R_2 c_1} + \frac{1-k}{R_2 c_2}$$

$$\Rightarrow \frac{w_0}{Q} = \frac{1}{c_1} + \frac{1}{c_1}$$

Or, $\frac{w_0}{Q} = \frac{2}{c_1}$

Also, $W_0 = \frac{1}{\sqrt{R_1 R_2 c_1 c_2}} = \frac{1}{\sqrt{c_1 c_2}}$

But, we take, in normalized case,

$W_0 = 1$

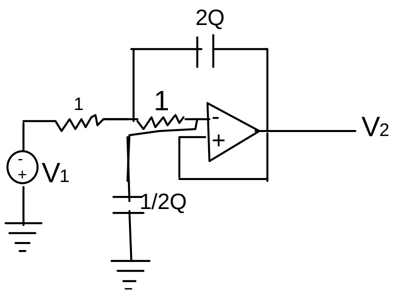
$\Rightarrow C_1 C_2 = 1$

$C_1 = 1/C_2$

Thus, $C_1 = 2Q$

$C_2 = 1/2Q$

$\therefore$ The final ckt will be,



Design –II (equal capacitance of equal feed bake):-

In this case,

$C_1 = C_2 = C = 1$

$R_A = R_B = R = 1$

$\therefore K = 1 + R_B/R_A = 2$

$\therefore W_0 = 1$

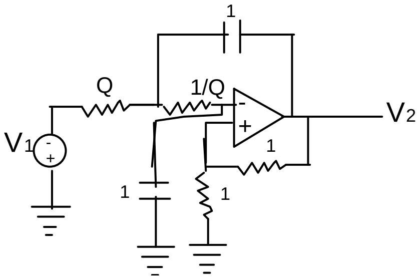
$$\frac{w_0}{Q} = \frac{1}{Q} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1-2}{R_2}$$

$\Rightarrow R_1 = Q$

Also, $W_0^{\,2} = \frac{1}{R_1 R_2 c_1 c_2} = 1$

$\Rightarrow R_2 = 1/Q$

$\therefore$ The final ckt will be,

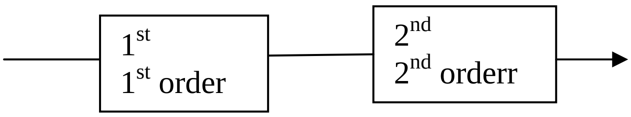


Example:- 01

Design a 4th order Butterworth filter using equal element of Sallen Key ckt. Then Let $w_0 = 2\pi 1000$ rad/sec & use capacitor of 0.1μF

Sol^n.:-

The 4th order Sallen Key in blocks can be represented by



From table,

$Q_1 = 0.54$ and $Q_2 = 1.31$

For 1st stage:-

$W_o = 1$, $Q_1 = 0.54$

& for equal element design in Sallen key,

$R_1 = R_2 = 1$

& $C_1 = C_2 = 1$

$R_A = 1$,

$R_B = 2 – 1/Q = 2 - 1/0.54$

$R_B = 0.148$

For 2nd stage:-

$W_O = 1$

$Q_2 = 1.31$

$R_1 = R_2 = R = 1$

$C_1 = C_2 = C = 1$

$R_A = 1$

$R_B = 1/Q_2 = 2-1/1.31 = 1.236$

$\therefore$ The design should be for,

$\Omega = 2\pi 1000$

& $C = 0.1\,\vartheta f$

We can apply both magnitude and frequency scaling at once.

Now, we know,

$$C_{new} = \frac{C_{old}}{C_{new}.kf} = \frac{1}{0.1\times10^{-6}\times 2\pi 1000} = 235.54$$

For 2nd stage,

$R_{Bnew} = K_m R_{Bold} = 1591.54*1.236 = 1967.14$

The ckt will be,

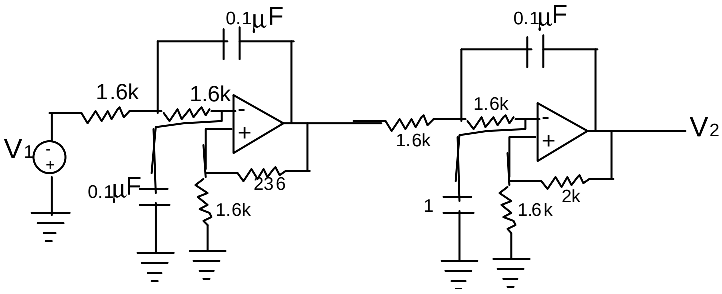


Fig: 4th order butter worth active Salleney biquad with equal element design for $W = 2\pi 1000\, rad/\sec$ & $C = 0.1\mu F$.

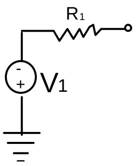
Gain adjustment (EqualiZation in Sallen key:-

$$T(S) = \frac{KWo^2}{S^2 + \left(\frac{Wo}{Q}\right).S + Wo^2} \quad \text{…………………….. (i)}$$

In Butterworth,

T (jo) = 1 or 0 d_B

But in equation (i) T (jo) = k (k>/1) which needs to be equalized.



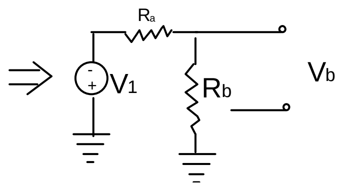


Fig:- i(a) fig:- i(b)

If ‘H’ is considered to be the gain provided by fig i(b) which is such that,

H. k = 1

Also it is to noted that, in Sallen key,

G = K

Also, H. G = 1

H =1/G

Now, T (S) = $\frac{V_b}{V_1} = \frac{R_b}{R_a + R_b}$

Also, we should remember that,

$$\frac{R_a R_b}{R_a + R_b} = R_1$$

Now, solving the above equation by setting R=1, we get R_a = 1/H

In term of ‘G’ the Value of R_a & R_b is

$$\boxed{Ra = G}$$

$$\boxed{R_b = \frac{G}{G-1}}$$

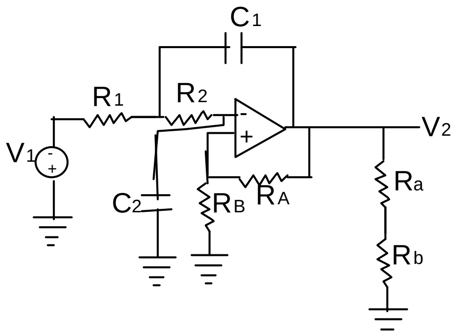
In term 'Q' R_a and R_b can be expressed as,

$$Ra = 3 - \frac{1}{Q}$$

$$\therefore G = K = \frac{3-1}{Q}$$

$$Rb = \frac{3Q-1}{2Q-1}$$

Gain Enhancement (Increment) in Sallen key:-



We have, gain,

$$K = 3 - \frac{1}{Q} = 1 + \frac{R_A}{R_B}$$

But, sometimes for given 'Q' the Value of gain will be Very small and amplification to our need. Although the separate ckt for gain enhancement can be used, the Sallen key ckt itself can be modified to compensate the gain, using additional arrangement of two resistor as, shown in the fig (ii)

Let, $C_1 = C_2 = C$

& $R_1 = R_2 = R$ then,

T(S) = of sallen key will be,

$$T(S) = \frac{\frac{k}{R^2C^2}}{S^2 + \left(\frac{3-\mu k}{RC}\right)S + \frac{1}{R^2C^2}}$$

Where, $\mu = \frac{R_b}{R_a + R_b}$

$$\therefore \frac{Wo}{Q} = \frac{3-\mu k}{RC}$$

$$Q = \frac{1}{3-\mu k}$$

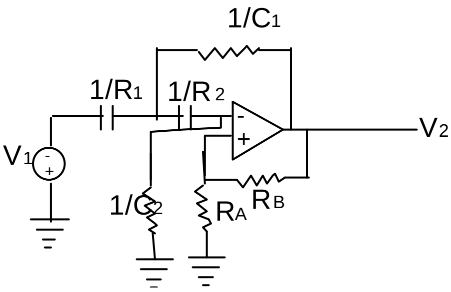
For a given Value of 'Q' the gain 'k' can be increased to our requirement by proportionally decreasing the new factor μ.

High pass sallen key:-

In this case,

$$T_{HP}(S) = \frac{GS^2}{S^2 + \left(\frac{Wo}{Q}\right).S + Wo^2}$$

Applying RC-CR transformation in active low pass Sallen key biquid, in the non-inverting terminals we get the following final ckt $T_{HP}(S)$ as,



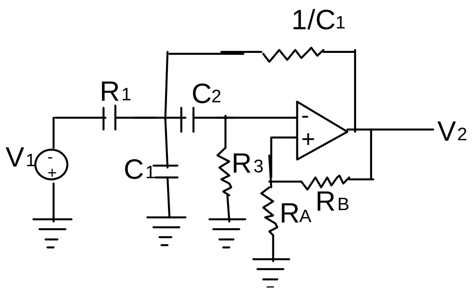
$$T_{HP(S)} = \frac{KS^2}{S^2 + \left(\frac{1}{R_2C_2} + \frac{1}{R_2C_1} + \frac{K-1}{C_1R_1}\right).S + \frac{1}{R_1R_2C_1C_2}}$$

Where, $W_O = \frac{1}{\sqrt{R_1R_2C_1C_2}}$

G = k

$$\frac{Wo}{Q} = \frac{1}{R_2C_2} + \frac{1}{R_2C_1} + \frac{k-1}{R_1C_1}$$

Band pass Sallen key Biquad:-



In this case,

$$TBP(S) = \frac{\left(\frac{k}{R_1C_1}\right).S}{S^2 + \left(\frac{1}{R_1C_1} + \frac{1}{R_3C_2} + \frac{1}{R_3C_1} + \frac{1-k}{R_2C_1}.S + \frac{R_1+R_2}{R_1R_2R_3C_1C_2}\right)}$$

Where,

$$W_O = \sqrt{\frac{R_1+R_2}{R_1R_2R_3C_1C_2}}$$

$$Q = \frac{W_O}{\frac{1}{R_1C_1} + \frac{1}{R_3C_2} + \frac{1}{R_3C_1} + \frac{1-k}{R_2C_1}}$$

$$G = \frac{\frac{k}{R_1C_1}}{\frac{1}{R_1C_1} + \frac{1}{R_3C_2} + \frac{1}{R_3C_1} + \frac{1-k}{R_2C_1}}$$

Also, in standard from,

$$TBP(S) = \frac{G\left(\frac{W_O}{Q}\right).S}{S^2 + \left(\frac{W_O}{Q}\right).S + WO^2}$$

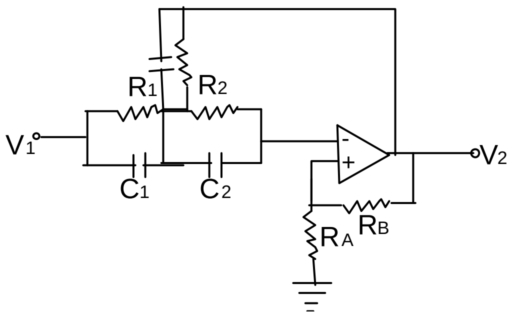
Where, $W_1 = W_O - Bw/2$

$W_2 = W_O + Bw/2$

And, $Q = \frac{W_O}{Bw} = \frac{W_O}{W_2 - W_1}$

Band stop Sallen key:-

$$T_{BS}(S) = \frac{G\left(S^2 + W_O{}^2\right)}{S2 + \left(\frac{W_O}{Q}\right).S + W_O{}^2}$$



Assuming,

$R_2 = R_1 = R$

$C_1 = C_2 = C$

$R_3 = R/2$

We obtained,

$$T_{BS}(S) = \frac{K\left(S^2 + \frac{1}{R^2C^2}\right)}{S^2 + \frac{4(1-k)S}{RC} + \frac{1}{R^2C^2}}$$

$$W_O = \frac{1}{RC}$$

$$Q = \frac{1}{4(1-k)}$$

$$G = K = 1 + \frac{R_B}{R_A}$$

Use equal amount design to obtain Bond pass Sallen key Biquad with W_O =1 & Q =10. Also find the upper band & lower band frequency. When W_O =1000Hz

W_O =1

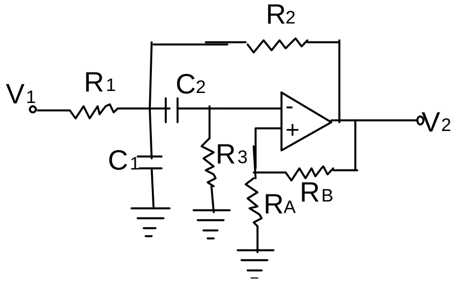
Q = 10

W_1, W_2 if W_O = 1000Hz

$$W_O = \sqrt{\frac{R_1 + R_2}{R_1 R_2 R_3 C_1 C_2}}$$

$$Q = \frac{W_O}{\frac{1}{R_1C_1} + \frac{1}{R_3C_2} + \frac{1}{R_3C_1} + \frac{1-k}{R_2C_1}}$$

$$G = \frac{\frac{k}{R_1C_1}}{\frac{1}{R_1C_1} + \frac{1}{R_3C_2} + \frac{1}{R_3C_1} + \frac{1-k}{R_2C_1}}$$



$R_1 = R_2 = R_3 = R$

$C_1 = C_2 = C = 1$

From which, we get,

$$W_O = \sqrt{\frac{2R}{R^B}} = \frac{\sqrt{2}}{R}$$

But,

$W_O = 1$

$\therefore R_W = \sqrt{2}$

Also, $10 = \frac{1}{\frac{4-k}{R}} \Rightarrow 10 = \frac{\sqrt{2}}{4-k} \Rightarrow 4-k = \frac{\sqrt{2}}{10} \Rightarrow K = 4 - \frac{\sqrt{2}}{10}$

From which,

$$K = 4 - \frac{R}{10} = 4 - \frac{\sqrt{2}}{10} = 3.86$$

Again,

$$G = \frac{k}{4-k} = \frac{3.86}{4-3.86} = 27.28$$

$\therefore G = -27.28$

$\therefore$ The required gain (K=3.86) for design parameter W_O = 1 & Q =10 is less than the gain (G = 27.28), so gain must be reduced. For this, we need the two resisters (R_a & R_b). Sampling by replacing R_1 so, that,

R_a =G =27.28

$R_b = \frac{G}{G-1} = \frac{27.28}{27.28-1} = 1.04$

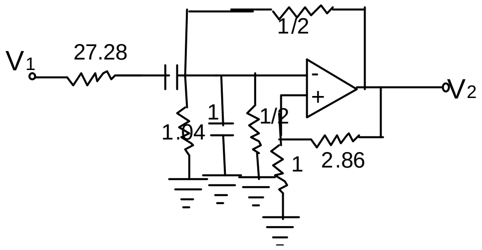
Again, We know,

$K = 1 + R_B/R_A$

For, $R_A = 1$

R_B = k-1 + 3.86 – 1 = 2.86

$\therefore$ The required final ckt will be:-



Now,

New frequency = 1000H_Z = Ω_0

$\therefore$ Frequency scaling factor kf = $\frac{\Omega_0}{w_0} = 1000$

$\therefore C_{new}$ = cold/kf

Or, C_{new} = 1/1000 = 1mf

$C_{1new} = C_{2new}$ = 1mf

Now for upper band and lower band frequency Ω_0 = Q/BW

$\Rightarrow$ BW = Q/ Ω_0 = 10/1000 = 0.01

$\Omega_1 = \Omega_0$ – BW/2 = 1000-0.01/2 = 999.999HZ

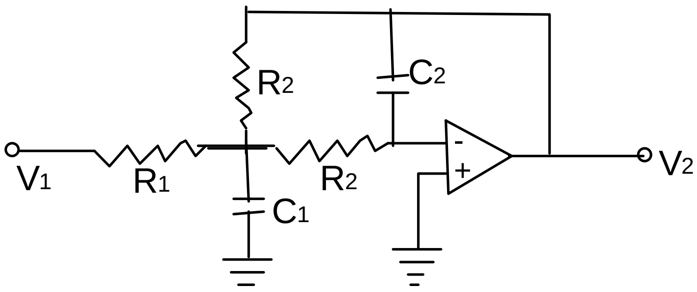
$\Omega_2 = \Omega_0$ + Bw/2 = 1000+0.01/2 = 1000.005HZ

Question:-.1 Design a 4th order butterworth active Sallen key law pass filter with unity gain. Realise it with practical components.

Question:-.2 Design a 5th order butterworth active Sallen key law pass filter with equal feed back resistance and equal capacitance Values. Then use, $w_0 = 2\pi$ 1000rad/sec and C =1μF .

Question:-.3 Design a 4th order buttterworth active Sallen Key low pass filter with equal element design.

Multiple feed back Biquad current:-



Fig(i) low pass MFB

$$T(s) = \frac{v_2}{v_1} = \frac{-\frac{1}{R_1R_3c_1c_2}}{s^2 + \frac{1}{c_1}\left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right)s + \frac{1}{R_2R_3c_1c_2}} \text{………(i)}$$

In standard form,

$$T(s) = \frac{-Gw_0{}^2}{s^2 + \left(\frac{w_0}{Q}\right)s + w_0{}^2} \text{……….(ii)}$$

Comparing eqn (i) & (ii)

$$w_0{}^2 = \frac{1}{R_2R_3c_1c_2}$$

$$\Rightarrow w_0 = \frac{1}{\sqrt{R_2R_3c_1c_2}}$$

And, $\frac{w_0}{Q} = \frac{1}{c_1}\left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right)$

and, $G = R_2/R_1$

Equating (i) can be modified in the form,

$$T(s) = \frac{v_2}{v_1} = -\frac{Gb_0}{s^2 + b_1s + b_0} \text{………..(iii)}$$

Where, $b_0 = W_0{}^2$

<u>Design in terms of bo & b_1</u>

Let, $C_1 = 1F$

$$\Rightarrow b_o = \frac{1}{R_2R_3C_2}$$

$$\Rightarrow G = \frac{R_2}{R_1}$$

$$\& \ b_1 = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

Solving,

$$R_2 = \left[\frac{2C_2bo}{b_1 + \sqrt{b_1{}^2 - 4C_2b_o(1+G)}}\right]^{-1}$$

<u>Example: 01</u>

Design a biquad ckt for $G = 5$, $b_1 = 1.2$ & b_O =1

Here, given,

G = 5

b_1 = 1.2

b_O =1

$bo = 1 \Rightarrow W_O = 1$

& we know,

$$b_1 = \frac{W_o}{Q}$$

$$\Rightarrow Q = \frac{W_O}{b_1} = \frac{1}{1.2} = 0.833$$

Also,

$$\frac{R_2}{R_1} = 5$$

Let, C_1= 1F (Choose higher)

C2 = 0.05F (Choose lower Value)

For which,

$$R_2 = \left[\frac{2 \times 0.05 \times 1}{1.2 + \sqrt{1.2^2 - 4 \times 0.05 \times 1(1+5)}}\right]^{-1}$$

=16.89

≈ 17

$\therefore R_2 = 17$

But,

$$5 = \frac{R_2}{R_1}$$

$$\Rightarrow R_1 = \frac{R_2}{5} = \frac{17}{5}$$

$\therefore R_1 = 3.4$

Again,

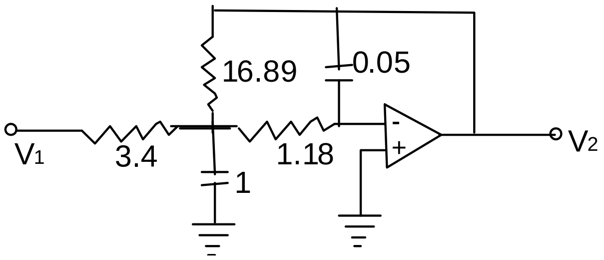
$$b_o = \frac{1}{R_2 R_3 C_2}$$

Or, $R_3 = \frac{1}{b_o R_2 C_2}$

$$= \frac{1}{1 \times 17 \times 0.05}$$

R_3 = 1.18

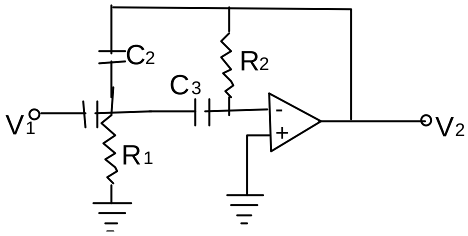
$\therefore$ The final design will be



It is to be noted that,

$$\frac{1}{C_1}\left(\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_3}\right)=1\left(\frac{1}{3.4}+\frac{1}{16.89}+\frac{1}{1.18}\right)=1.2=b_1$$

High pass MFB:-



In this case,

$$\text{T(S)} = \frac{\frac{C_1}{C_2}.S^2}{S^2+\frac{C_1+C_2+C_3}{R_2C_2C_3}+\frac{1}{R_1R_2C_2C_3}}$$

The standard form is,

$$\text{T(S)} = \frac{-GS^2}{S^2+\left(\frac{W_O}{Q}\right).S+WO^2} = \frac{-GS^2}{S^2+b_1s+b_O} \quad \text{................ (ii)}$$

With, $C_1 = C_3 = 1F$

$G = C_1 / C_2 = 1/C_2$

$\therefore C_2 = 1/G$

Also,

$$b_O = \frac{1}{R_1R_2C_2C_3}$$

$$b_1 = \frac{C_1+C_2+C_3}{R_2C_2C_3}$$

Now, b_o can rewritten as,

$$b_1 = \frac{2+\frac{1}{G}}{\frac{R_2}{G}}$$

$$\frac{R_2b_1}{G} = R_2 = \frac{\left(2+\frac{1}{G}\right)S}{b_1}$$

$$R_2 = \left(\frac{2G+1}{b}\right) \text{ .... (iV)}$$

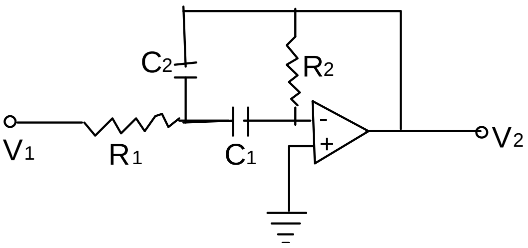
Similarly,

$$b_O = \frac{1}{R_l R_2 C_2 C_3} = \frac{1}{R_1 \frac{(2G+1)}{b_1} \cdot \frac{1}{G} .1} = \frac{G}{R_1(2G+1)}$$

$$\therefore R_1 = \frac{Gb_1}{b_0(2G+1)} \text{ ....(V)}$$

Equation (iii), (iV) & (V) show that the component Value (with $C_1 = C_2 = 1$) can be adjusted from the design parameters G, b_0 and b_2

Band pass MFB:-



$$T(s) = \frac{-\frac{1}{R_1c_2}.s}{s^2 + \left(\frac{1}{R_2c_1} + \frac{1}{R_2c_2}\right)s + \frac{1}{R_1R_2c_1c_2}}$$

Where,

$$w_0{}^2 = \frac{1}{R_1R_2c_1c_2} = b_0$$

$$\frac{w_0}{Q} = \frac{1}{R_2c_1} + \frac{1}{R_2c_2} = b_0 \Rightarrow Q = \frac{\sqrt{R_1R_2}}{\sqrt{\frac{c_2}{c_1}} + \sqrt{\frac{c_1}{c_2}}}$$

$$G = \frac{R_2c_1}{R_1(c_1 + c_2)}$$

It is to be noted that,

$$Q = \frac{w_0}{Bw} = \frac{w_0}{w_2 - w_1}$$

Where, $w_1 = w_0 - \frac{Bw}{2}$

And $w_2 = w_0 + \frac{Bw}{2}$

* Band step filter for MFB (आफै गर्ने)

KHN Biquad (kerwin – Huelsman-Newcomb):-

The general KHN Biquad ckt is given by;

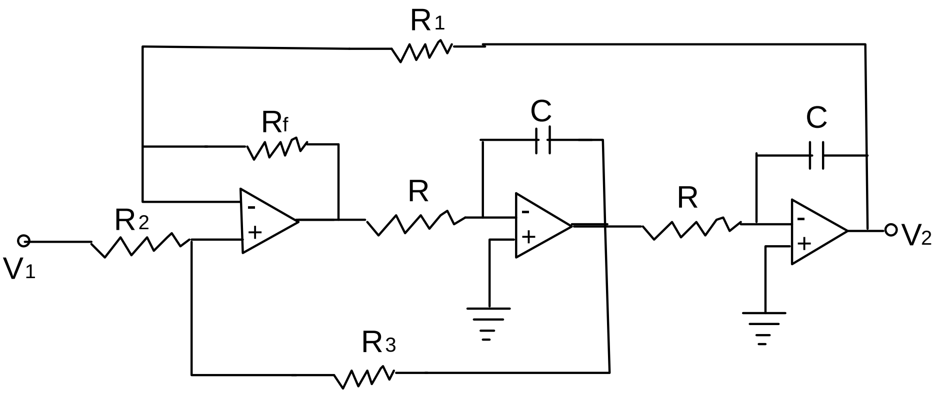


Fig (i) universal KHN-Biquid

If we consider only high pass, the ckt will be as follows:

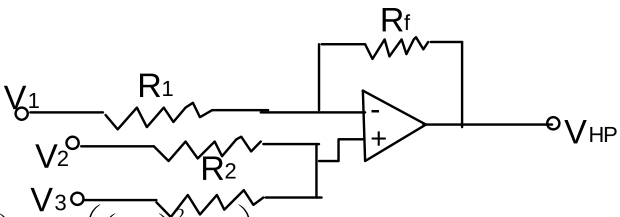


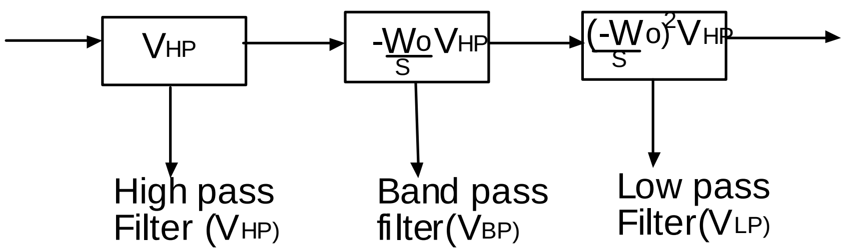
Fig (ii) high passes KHN Biquad

The o/p for fig (ii) will be,

$$V_{HP} = \frac{R_3}{R_2+R_3}\left(1+\frac{Rf}{R_1}\right)v_1 + \frac{R_2}{R_2+R_3}\left(1+\frac{Rf}{R_1}\right)\left(-\frac{w_0}{s}V_{HP}\right) - RF\left(\left(\frac{w_0}{s}\right)^2 V_{HP}\right)$$

$$\left[\sin ce, V_{BP} = -\frac{1}{Rcs}.V_{HP} - \frac{w_0}{s}V_{HP} \;\&\; V_{LP} = -\frac{1}{Rcs}V_{BP} = \left(-\frac{1}{Rcs}\right)^2 V_{HP} = \left(-\frac{w_0}{s}\right)^2 V_{HP}\right]$$

Thus in general, blocks,



But the standard form of KHN is giVen by, (for the 1st stage)

$$V_{HP} = kv_1 - \frac{1}{Q}\frac{w_0}{s}V_{HP} - \left(\frac{w_0}{s}\right)^2 V_{HP}$$

Where,

$Rf/R_1 = 1$

$R_3/R_2 = 2Q-1$

$K = 2- 1/Q$

$W_0 = 1/Rc$

KHN Biquad ckt is also called universal biquad ckt because from its Various stages as shown in fig (i), low pass prototype, high pass prototype and band pass prototype can be achieve from a single ckt.

Chapter:-9

Sensitivity:-

Let us consider the following two fig (i) & (ii) with

$L_1 = 0.9956H$
$C_1 = 0.91097F$
$R_1 = 1\Omega$,
$K = 1\ \Omega$
$R_3 = 1.0143\ \Omega$
$R_4 = 8.9422\ \Omega$
$C_2 = 0.1F$
$C_3 = 1F$

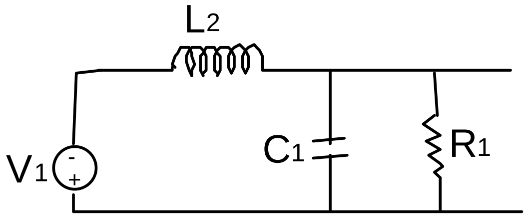


Fig:- (i)

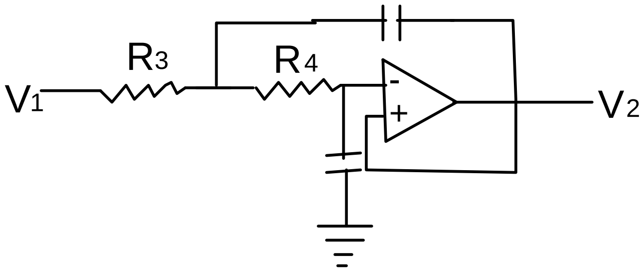


Fig:- (ii)

The transfer function of both the ckts are same which is $T(s) = \dfrac{1.10251}{s^2 + 1.09735 + 1.10251}$

At W = 0, i.e. $T(y_o) = 1$ which indicates that both gives the Butterworth response.

Now, let us assume that all elements are increased by 1%

For passive $|T(y_0)| = 0.99168 \rightarrow$ gain reduced 0.83%

& for-active, $|T(y_0)| = 0.98308 \rightarrow$ gain reduced 1.7%

Thus, it proves that passive filters are less sensitive to element changing than active filters.

Definition of sensitivity:-

If x is the element & y is the design parameter for example R may be element & w_0 may be the design parameter then sensitivity is denoted by $\int_x^y$ defined by, $\int_x^y = \dfrac{\%changeiny}{\%changeinx} = \dfrac{\frac{\Delta y}{y}}{\frac{\Delta x}{x}} = \dfrac{x}{y}.\dfrac{\Delta y}{\Delta x}$

$$\int_x^y = \frac{x}{y}\frac{dy}{dx}$$

$\Rightarrow$ If S = 2 then 1% change in x, result 2% change in y.

$\Rightarrow$ If S = 0.1 than 1% change in x, result 0.1% change in y.

* Sensitivity is known $\Rightarrow$ single parameter sensitivity
  - $\Rightarrow$ First order sensitivity
  - $\Rightarrow$ Differential sensitivity
  - $\Rightarrow$ Classical sensitivity
  - $\Rightarrow$ Bode sensitivity

* Properties of first order sensitivity:-

(i) $\int_x^y = \dfrac{x}{ky}\dfrac{dky}{dx} = \dfrac{x}{y}\dfrac{dy}{dx} = \int_x^y$

$$\therefore \int_x^{ky} = \int_x^{y} \quad \text{Where, k = constant}$$

(ii) $\int_x^{y+k} = \frac{x}{y+k}\frac{d(k+y)}{dx} = \frac{x}{y+k}\frac{dy}{dx} = \frac{x}{y}\frac{dy}{dx}\frac{y}{y+k}$

$$\therefore \int_x^{y+k} = \frac{y}{y+k}\int_x^{y}$$

(iii) $\int_{\frac{1}{x}}^{y} = \frac{\frac{1}{x}}{y}.\frac{dy}{d\left(\frac{1}{x}\right)} = \frac{1}{xy}.\frac{dy}{d(x)^{-1}} = \frac{1}{xy}.\frac{dy}{\frac{dx^{-1}}{dx}.dx}$

Or, $\int_{\frac{1}{x}}^{y} = \frac{1}{xy}\frac{dy}{dx}.\frac{1}{-x^{-2}} = -\frac{1}{x^{-1}y}\frac{dy}{dx} = -\frac{x}{y.\frac{dy}{dx}}$

$$\therefore \int_{\frac{1}{x}}^{y} = -\int_x^{y}$$

(iV) $\int_x^{\frac{1}{y}} = -\int_x^{y}$

(V) $\int_x^{y_1 y_2} = \int_x^{y_1} + \int_x^{y_2}$

(Vi) $\int_{xn}^{y} = \frac{1}{n}\int_x^{y}$

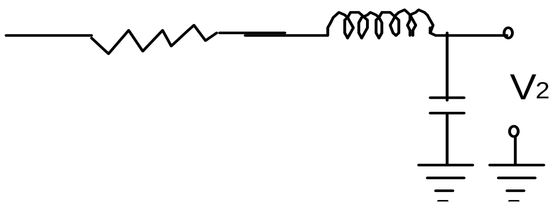
(Vii) $\int_x^{\frac{y_1}{y_2}} = \int_x^{y_1} - \int_x^{y_2}$

(ix) $\int_x^{\ln y} = \frac{1}{\ln y}\int_x^{y}$

(x) $\int_x^{\exp(y)} = y\int_x^{y}$

*** Derive all the properties**

Sensitivity of passive ckt (Biquad):-



We know,

$$T(s) = \frac{Gw_0^{\ 2}}{s^2 + \left(\frac{w_0}{Q}\right)s + w_0^{\ 2}}$$

We need to find out,

$$S_R^{\ w_0}, S_L^{\ w_0}, S_R^{\ Q}, S^Q{}_L, S^Q{}_c$$

from figure,

$$T(s) = \frac{\frac{1}{Lc}}{s^2 + \frac{R}{Ls} + \frac{1}{Lc}}$$

Comparing, $w_0 = \frac{1}{\sqrt{Lc}} = \frac{1}{L^{\frac{1}{2}}C^{\frac{1}{2}}} = L^{-\frac{1}{2}}.C^{-\frac{1}{2}}$

Or, $\frac{w_0}{Q} = \frac{R}{L}$

$$\Rightarrow Q = \frac{L.w_0}{R} = \frac{L.L^{-\frac{1}{2}}.C^{-\frac{1}{2}}}{R} = L^{\frac{1}{2}}C^{-\frac{1}{2}}R^{-1}$$

(1) $S_R^{\ w_0} = \frac{R}{w_0}.\frac{d(w_0)}{dR} = \frac{R}{L^{-\frac{1}{2}}C^{-\frac{1}{2}}}.\frac{d}{dR}\left(L^{-\frac{1}{2}}C^{-\frac{1}{2}}\right) = 0$

(2) $S_R^{\ Q} = \frac{R}{Q}.\frac{d(Q)}{dR} = \frac{R}{L^{\frac{1}{2}}C^{-\frac{1}{2}R^{-1}}}.\frac{d\left(L^{\frac{1}{2}}C^{-\frac{1}{2}}R^{-1}\right)}{dR} = -1$

(3) $S^{W_0}{}_Q = -\frac{1}{2}$

(4) $S^Q{}_L = \frac{1}{2}$

(5) $S_c^{\ w_0} = -\frac{1}{2}$ (6) $S_c^{\ Q} = -\frac{1}{2}$

Sensitivity of active Biquad ckt:-

(1) Two Thomas Biquid

In this case,

$$w_0 = \frac{1}{\sqrt{R_2R_4c_1c_2}} = R_2^{-\frac{1}{2}}R_4^{-\frac{1}{2}}c_1^{-\frac{1}{2}}c_2^{-\frac{1}{2}}$$

$$Q = \sqrt{\frac{R_1^{\ 2}c_1}{R_2R_4R_2}} = R_1R_2^{-\frac{1}{2}}R_4^{-\frac{1}{2}}c_1^{\frac{1}{2}}c_2^{-\frac{1}{2}}$$

$$G = R_2R_3^{-1}$$

1) $S_{Rl}^{W_O} = \frac{-1}{2}$

2) $S_{R_4}^{W_O} = \frac{-1}{2}$

3) $S_{C_1}^{W_O} = \frac{-1}{2}$ $S_{R_2,R_4,C_1,C_2}^{W_O} = \frac{-1}{2}$

4) $S_{C_2}^{W_O} = \frac{-1}{2}$

Similarly,

5) $S_{R_1}^{Q} = 1$

6) $S_{R_2}^{Q} = -\frac{1}{2}$

7) $S_{R_1}^{Q} = -\frac{1}{2}$

8) $S_{C_1}^{Q} = +\frac{1}{2}$

9) $S_{C_2}^{Q} = -\frac{1}{2}$

10) $S_{R_2}^{G} = 1$

11) $S_{R_3}^{G} = -1$

(1) $S_{R_2}^{W_O} = -\frac{1}{2} = \frac{R_2}{W_O}.\frac{d(W_O)}{d(R_2)}$

$$= \frac{R_2}{W_O}.\frac{d\left(R_2^{\frac{-1}{2}}.R_4^{\frac{-1}{2}}.C_1^{\frac{-1}{2}}.C_2^{-\frac{1}{2}}\right)}{d(R_2)}$$

$$= \frac{R_2}{W_O}.-\frac{1}{2}.R_2^{-\frac{3}{2}}.R_4^{-\frac{1}{2}}.C_1^{-\frac{1}{2}}.C_2^{-\frac{1}{2}}$$

$$= \frac{R_2.-\frac{1}{2}.R_2^{-\frac{3}{2}}.R_4^{-\frac{1}{2}}.C_1^{-\frac{1}{2}}.C_2^{-\frac{1}{2}}}{R_2^{-\frac{1}{2}}.R_4^{-\frac{1}{2}}.C_1^{-\frac{1}{2}}.C_2^{-\frac{1}{2}}}$$

$$= \frac{\frac{-1}{2}.R_2^{-\frac{3}{2}}}{R_2^{-\frac{3}{2}}} = \frac{-1}{2}$$

(2) **<u>Sallen key Biquid ckt</u>**:-

In this case,

$$W_O = \frac{1}{\sqrt{R_1R_2C_2C_1}}$$

$$Q = \frac{\frac{1}{\sqrt{R_1R_2C_1C_2}}}{\frac{1}{R_1C_1}+\frac{1}{R_2C_1}+\frac{(1-K)}{R_2C_2}}$$

$$K = 1 + \frac{R_B}{R_A}$$

Sensitivity	**Equal Element design** $K = 3 - \frac{1}{Q}$ ……. (1)	**Equal capacitance and feedback resistance** (K = 2) ………… (2)
1) $S_{R_1}^{Q}$	$-\frac{1}{2}+Q$	$-\frac{1}{2}+Q$
2) $S_{R_2}^{Q}$	$+\frac{1}{2}-Q$	$+\frac{1}{2}-Q$
3) $S_{C_1}^{Q}$	$-\frac{1}{2}+2Q$	$\frac{1}{2}+Q$
4) $S_{C_2}^{Q}$	$\frac{1}{2}-2Q$	$-\frac{1}{2}-Q$
5) S_{k}^{Q}	3Q-1	2Q
6) $S_{R_A}^{Q}$	1-2Q	-1
7) $S_{R_B}^{Q}$	2Q-1	1
8) $S_{R_A}^{k}$	$\frac{(-2Q-1)}{(3Q-1)}$	$-\frac{1}{2}$
9) $S_{R_B}^{k}$	$\frac{(-2Q-1)}{(3Q-1)}$	$\frac{1}{2}$
	High sensitive	Moderately sensitive

(1) Design (1) is the simplest implementation interim of element Values out it's disadvantage is that it is highly.

(2) Design (2) is less sensitive them design (1) in sensitivity is achieved at the expense of large resistance Value spread.

(3) Design (3) is the least sensitivity is achieved at the expense of large capacitor Value spread.

Q. Derive $S_{R_1}^{Q} = -\frac{1}{2} + Q$ in Sallen key equal amount design.

Multiparameter sensitiVity:-

Let,

Y = f (x_1, x_2, x_3 ……….. x_n)

Then,

$$\frac{\Delta y}{y} = \sum_{i=1}^{n} S_{x_i}^{y} \frac{\Delta x_i}{x_i}$$

Let, R_1, R_2 ………… R_n be the receptivity group and C_1, C_2, C_n be the capacitive group and $\mu_1, \mu_2, ...\mu_n$ be the capacitive gains.

Then,

$$\frac{\Delta y}{y} = \left(\sum_{i=1}^{m} S_{R_i}^{y} \right) \frac{\Delta R}{R} + \left(\sum_{i=1}^{n} S_{C_i}^{y} \right) \frac{\Delta C}{C} + \left(\sum_{i=1}^{k} S_{\mu_i}^{y} \right) \frac{\Delta \mu}{\mu} \quad \text{………… (i)}$$

Thus, we can define the multiparameter sensitivity as the combined effect of all the individual sensitivity in a particular ckt and is generally express as shown in eqn (i).

Chapter:-10

Higher order Active filter:-

The higher order active filter, in terms of transform function, can be defined by,

$$T(S) = \frac{a_m s^m + a_{m-1} s^{m-1} + \ldots\ldots\ldots + a_1 s + a_o}{b_n s^n + b_{n-1} s^{n-1} + \ldots\ldots\ldots + b_1 + b_o}$$

If 'n' is even, then, the higher order active filter in cascade realization may be expressed as:

$$T(S) = \prod_{i=1}^{n/2} \left(\frac{a_{2i} s^2 + a_{1i} s + a_{oi}}{b_{2i} s^2 + b_{1i} s + b_{oi}} \right)$$

Similarly, if 'n' is odd, then,

$$T(S) = \frac{a_u s + a_{o1}}{b_{11} s + b_{o1}} \prod^{(n+1)/2} \left(\frac{a_{2i} s^2 + a_{1i} s + a_{oi}}{b_{2i} s^2 + b_{1i} s + b_{o1}} \right)$$

Example 01:-

Design a low pass Butterworth active Sallen key filter with unity Voltage gain. The design filter ckt must meet the following specification.

$$\alpha_{\max} = 0.5 d_3$$

$$\alpha_{\min} = 10 d_3$$

w_p = 1000rad/sec

w_s = 200rad/sec

Choose appropriate element Values so that the filter can be practically realized.

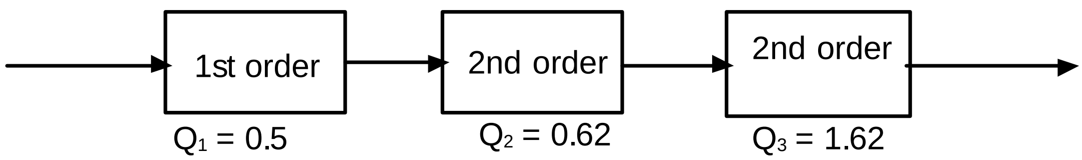
Soln:- The order of Butterworth filter is given by

$$n = \frac{\log\left[\left(10\alpha \frac{\max}{10} - 1\right)\right]}{}$$

= 4.83

= 5

$\therefore$ The filter is of 5^{th} order (i.e. n = 5)

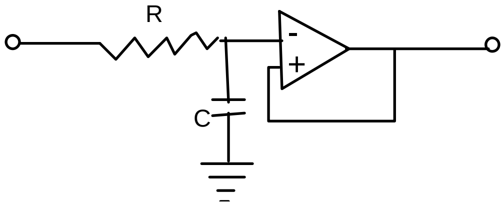


Also $w_0 = 1$

For given condition

$$\Omega_0 = \frac{wp}{(10)}$$

For 1st stage:-



Here, T(s) = $\dfrac{\frac{1}{Rc}}{s+\frac{1}{Rc}}$

$\therefore$ w_0 =1

1/Rc =1

Or, Rc = 1

Let, C_{new} = 0.1 μF

Again we also need to perform frequency scaling with

Kf = $\dfrac{\Omega_0}{w_0} = 1263.2$

$\therefore$ Applying both magnitude and frequency scaling

$C_{new} = \dfrac{cold}{kf.km}$

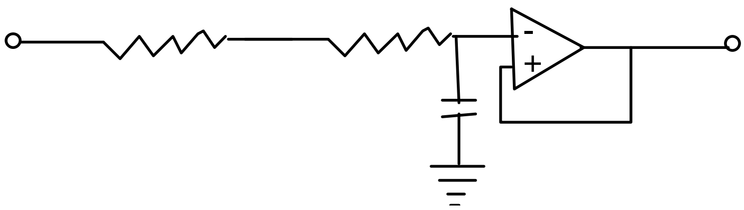
Or, 0.1 $\mu F = \dfrac{1}{1263.2 \times km}$

$\Rightarrow km = 7616.40$

R_{new} = km Rold

= 7916.40 X 1 = 7.916k

2nd stage:-



In unity Voltage gain of Sallen key

W_0 = 1

& R_1, R_2 = 1

C_1 = 2Q

C_2 = 1/2Q

Where, Q = 0.62

$\therefore C_1$ = 1.24F

C_2 = 0.806F

Applying magnitude and frequency scaling

Let, C_{1new} = 0.1 μF

Or, C_{1new} = C_1 old/kmkf

$\Rightarrow$ Km = 9816.34

$\therefore$ $C_{2new} = \dfrac{c_2 old}{kmkf} = \dfrac{0.806}{9816.34 \times 1263.2} = 0.065\mu F = 65.03nF$

Similarly,

$R_{1new} = R_{2new} = kmRold = 9816.34 \times 1 = 9.8$

For 3rd stage:-

$w_o = 1$

$R_1 = R_2 = 1$

$C_1 = 2Q$, $C_2 = 1/2Q$

For, Q = 1.62

$C_1 = 2Q = 2 X 1.62 = 3.24F$

$C_2 = 1/2Q = 1/2X1.62 = 0.308F$

For let $C_{1new} = 0.1\,\mu$ F

$$C_{1new} = \frac{Cold}{kf.km} = \frac{3.24}{0.1 \times 10^{-6} \times 1263.2} = 25649.14$$

$$C_{2new} = \frac{C_2 old}{km.kf} = \frac{0.308}{25649.14 \times 1263.2} = 9.51nF$$

$$\therefore R_{1new} = R_{2new} = kmR_1 old = 25649 \times 1 = 25.64k$$

Therefore the final ckt will be,

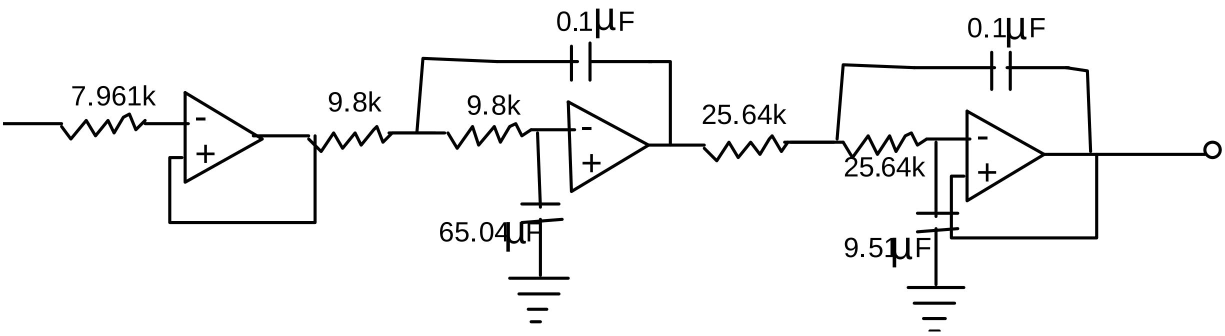


Fig: - 5th order low pass Butterworth active Sallen key Biquad with unity Voltage gain.

Ex: 02; Design a 5th order low pass Butterworth filter with F0 = 1 kHz and capacitance of Of 0.1 μ F. Implement this ckt in MFB.

Example:- 03

implement the same in two Thomas.

Example:- 04

In some application filter ckt must meet the following specification.

$$\alpha_{max} = 0.5dB$$

$$\alpha_{min} = 20dB$$

$$\frac{ws}{wp} = 2$$

The design must be highest sensitivity low pass Butterworth active Sallen key filter with unity Voltage gain. Chose appropriate element Values so that the filter can be realized.

Also calculate the sensitivity of Q with 1% incorrect in Values of filter elements due to some reason.

Chapter: - 11

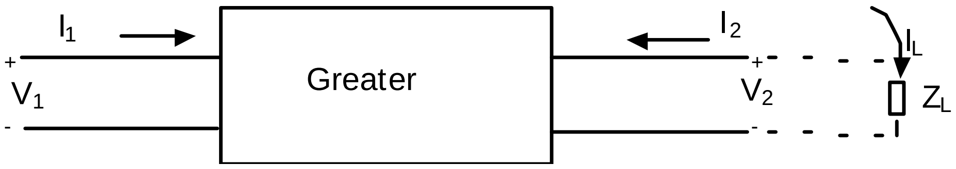
Simulation of passive n/w: -

Tellegen proposed a mode called gyrator where.

$V_1 = kI_2$............(i)

$V_2 = kI_1$………..(ii)

For the fig (i) shown below



Here, k is a real constant

Now, let Z_L be the load, then,

$V_2 = Z_L I_L$

Or, $V_2 = -Z_L K_L$

From equation (i) and (ii)

$$V_1 = k\left(\frac{v_2}{Z_L}\right)$$

Or, $V_1 = k\left(-\frac{(-kI_1)}{Z_L}\right)$ (from eqn (ii))

Or, $\frac{v_1}{I_1} = \frac{k^2}{Z_L}$

$\therefore Zin = \frac{K^2}{Z_L} = Z_{i1}$

If ZL is capacitor, then,

$Z_L = \frac{1}{cs}$

$\therefore Z_{in} = k_2 cs = L_{eq}.s$

Where,

$L_{eq} = k^2 c$

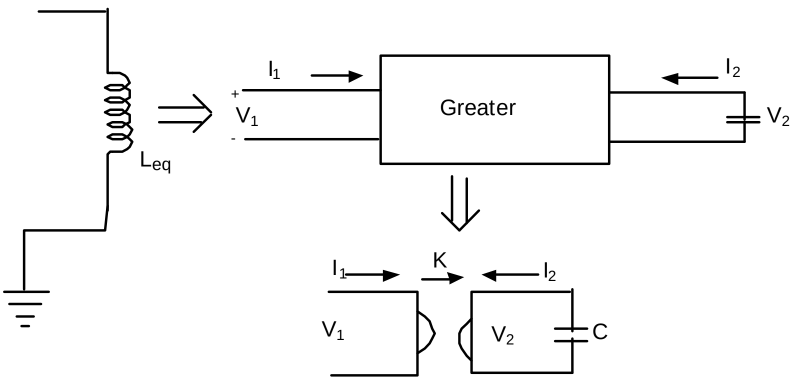


Fig:-(iii) symbol for Gaygrator

GIC (General Impedance Converter):-

It was developed by Antoniou, So also called Antoniou GIC.

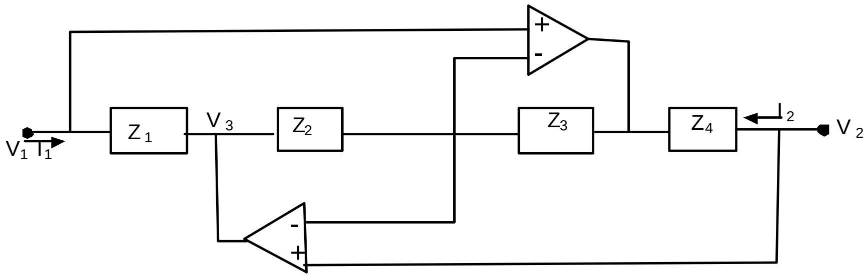


Fig (i) General impedance converter

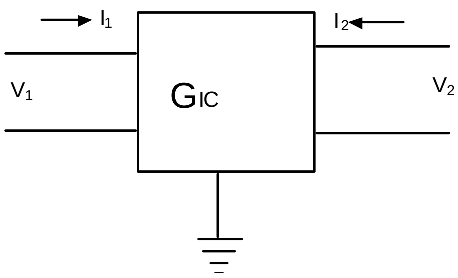


Fig:- (ii) symbol for Gic

It is to be noted that,

$V_1 = V_2$

& $I_1 = -I_2$

From fig (i)

$$I_1 = \frac{v_1 - v_3}{z_1}$$

$$I_2 = \frac{v_2 - v_4}{z_4}$$

$$= \frac{v_1 - v_4}{z_4}[\therefore v_1 = v_2]$$

Also, $I_1 = -I_2$

Or, $\frac{v_3 - v_1}{z_2} = -\left(\frac{v_4 - v_1}{z_3}\right)$

Or, $-\frac{I_1 z_1}{z_2} = \frac{I_1 z_4}{z_3}$

Or, $I_1 = \frac{-z_2 z_4}{z_1 z_3}.I_2$

We know,

$$Z_{in} = \frac{v_1}{I_1} = \frac{-z_1 z_3}{z_2 z_4}.\frac{v_2}{I_2}[\therefore v_1 = v_2]$$

$$= -k\frac{v_2}{I_2}$$

Where,

$k = \frac{z_1 z_3}{z_2 z_4}$

Let, Z_L be the lead, then,

$v_2 = Z_L I_L = -Z_L I_2$

Or, $\frac{v_2}{I_2} = -Z_L$

Thus,

$Z_{in} = kZ_L$

$Z_{in} = \frac{z_1 z_3}{z_2 z_4}.Z_L = Leq.S$

Always,

$Z_L = R_L$

$Z_1 = R_1$

$Z_3 = R_3$

Now, if $z_2 = \frac{1}{c_2 s}$ and $Z_4 = R_4$ then,

$z_{in} = \frac{R_1 R_3 c_2}{R_4}.R_L.s = Leq.S$

In this case, k = $\frac{R_1 R_3 C_4}{R_2}$

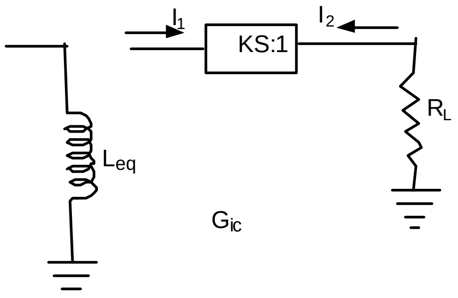
Also, $I_1 = \frac{-z_2 z_4}{z_1 z_3} I_2$

Or, $I_1 = -\frac{1}{ks}.I_2$

Or, $I_2 = -ksI_1$

$\Rightarrow -I_2 : I_1 = ks : 1$

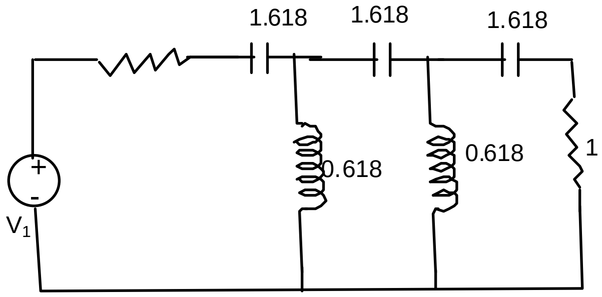
Thus the representation will be



It is to be noted that this is the case of Grounded inductor simulation.

Example:- 01

Simulate the following ladder ckt with GIC.



Soln.

We need to simulate L_2 & L_4 with GIC

For, L_2

We know that,

$L_{2eq} = KR_L$

Let, $Z_2 = 1/c_2s$

& $Z_1 = R_1$

$Z_3 = R_3$

$Z_4 = R_4$

If $R_1 = R_2 = R_4 = 1$

& $C_2 = 1$, then,

$L_{2eq} = R_L$

$\therefore R_L = L_{2eq} = 0.618$

$\therefore R_L = 0.618$

Similarly, for L_4

$R_L = 0.618$

$\therefore$ The final ckt will be,

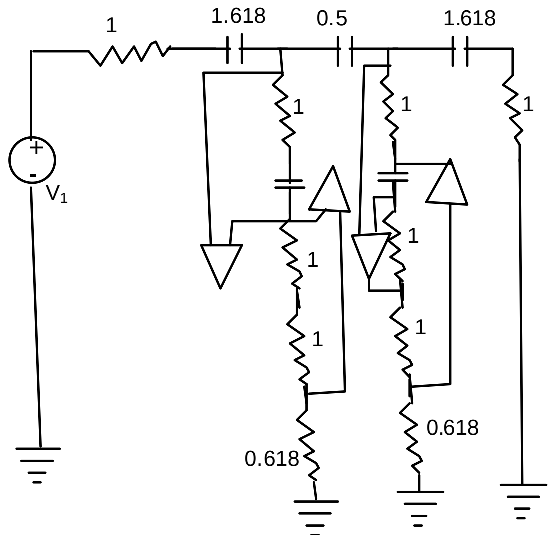
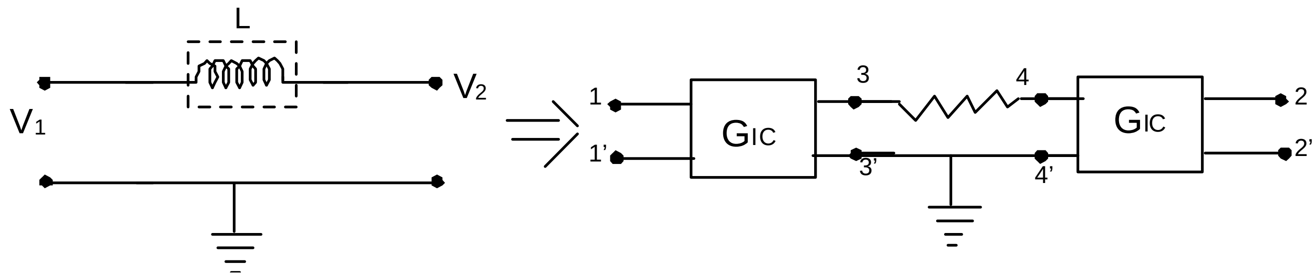
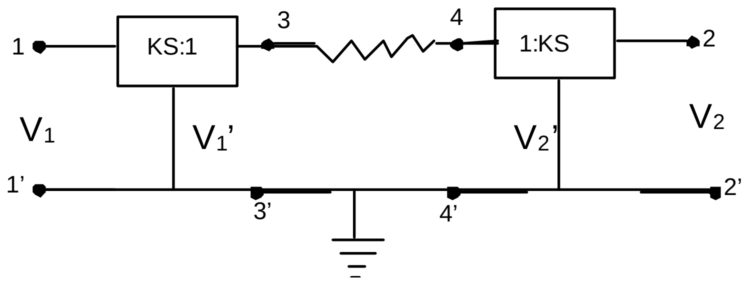


Fig: Equivalent ckt with GIC.

Floating Inductor simulator:-





From fig 1(a)

$$\frac{V_1 - V_2}{I_1} = L_{eq}.s \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(1)}$$

From fig (a) (c),

$$V_1 = V_1^1$$

$$V_2 = V_2^1$$

Also,

$$I_1^1 = -ksI_1$$

$$I_2^1 = -ksI_2 \quad \ldots\ldots\ldots\ldots\ldots \textit{(ii)}$$

$$I_1^1 = -I_2^1$$

$$I_1 = -I_2 \quad \ldots\ldots\ldots\ldots\ldots \textit{(iii)}$$

From eqn (ii) & (iii)

$$I_2^1 = KSI_1$$

Also,

$$\frac{V_1^1 - V_2^1}{I_2^1} = R$$

Or, $\dfrac{V_1 - V_2}{KSI_1} = R$

Or, $\dfrac{V_1 - V_2}{I_1} = KRS$ …………..(iV)

From eqn (i) & (iV),

Leq = K.R

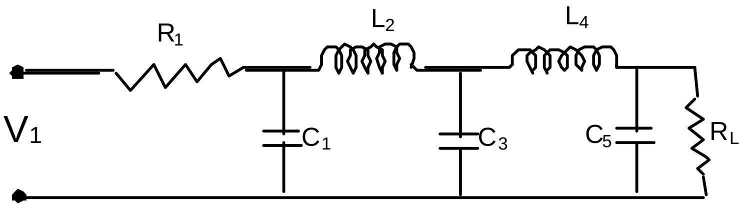
$$\boxed{R = \frac{Leq}{K}} \quad \ldots\ldots\ldots\ldots\ldots \text{(V)}$$

Where,

$$K = \frac{R_1R_2C_2}{R_4} \;\; or, \;\; \frac{R_1R_3C_4}{R_2}$$

Example:- 01

Simulate the following ladder ckt with GIC.



Ladder Design with frequency dependent –Ve resistor (FDNR):-

We know,

$$Z_{in} = \frac{Z_1Z_3}{Z_2Z_4}.Z_L \;\ldots\ldots\ldots\ldots\ldots\ldots\text{.. (i)}$$

If,

$$Z_1 = \frac{1}{c_1s}$$

$$Z_3 = \frac{1}{C_3S}$$

$Z_L = R_L$

$Z_2 = R_2$

$Z_4 = R_4$

Then, eqn (i) becomes,

$$z_{in} = \frac{\frac{1}{c_1s}.\frac{1}{c_3s}}{R_2R_4}.R_L$$

$$= \frac{R_L}{R_2R_4c_1c_3s^2}$$

$$\therefore z_{in} = \frac{1}{Ds^2} \;\ldots\ldots\ldots\text{(ii)}$$

Where,

$$D = \frac{R_2R_4c_1c_3}{R_L}$$

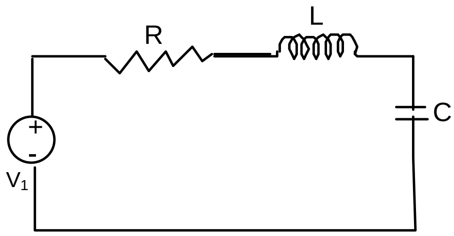
Put, s = jw

Then, $z_{in} = \frac{-1}{Dw^2}$ ………(iii)

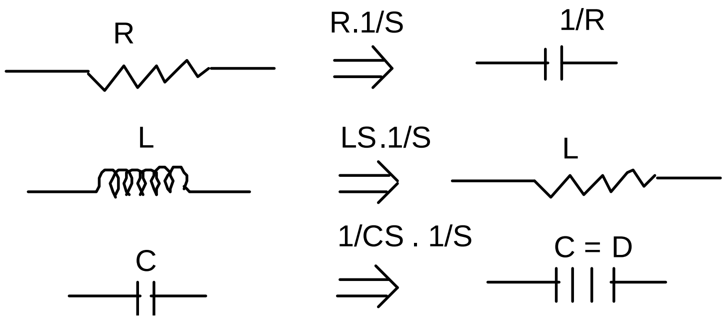
Equation (iii) define FDNR

Process,

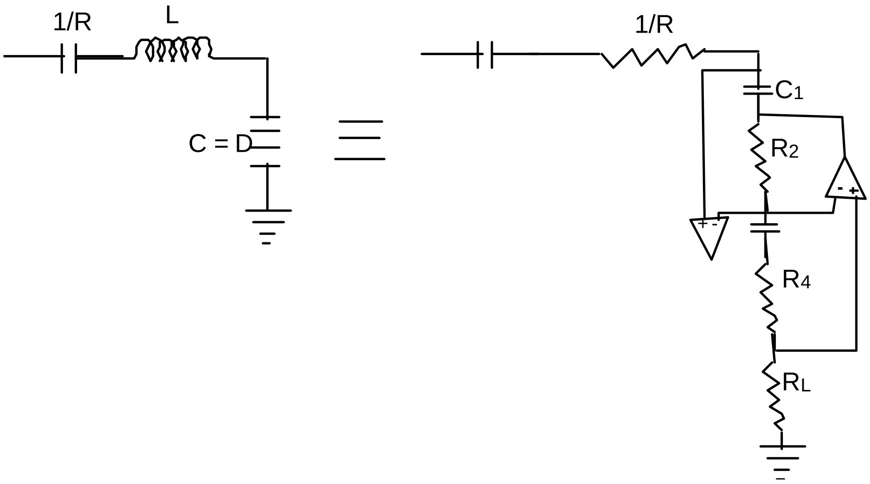
Let us consider the following simple RLC ckt.



We scale this ckt by 1/s i.e



∴ The final ckt will be,



Here,

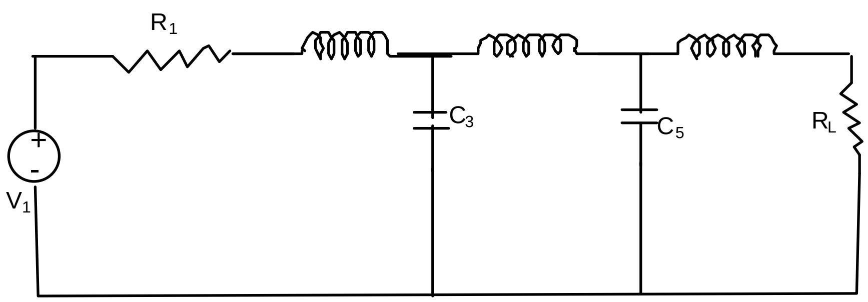
$$D = \frac{R_2 R_4 c_1 c_3}{R_L}$$

If, $R_2 = R_4 = 1$ & $c_3 = c_3 = 1$

Then,

$$R_L = \frac{1}{D} = \frac{1}{c}$$

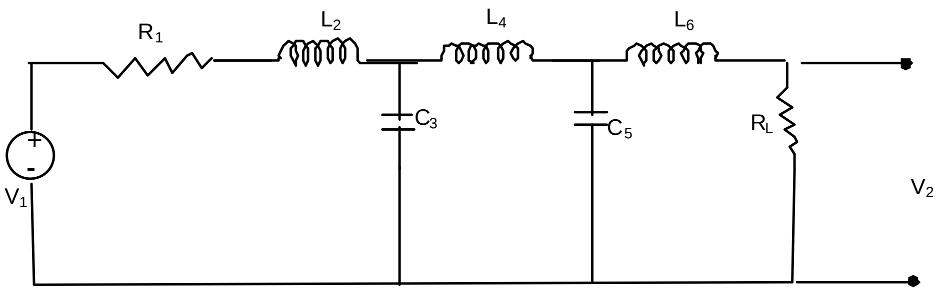
Example:-02



Realize the above ckt

Leapfrog simulation of ladders:-

Consider a ladder circuit,



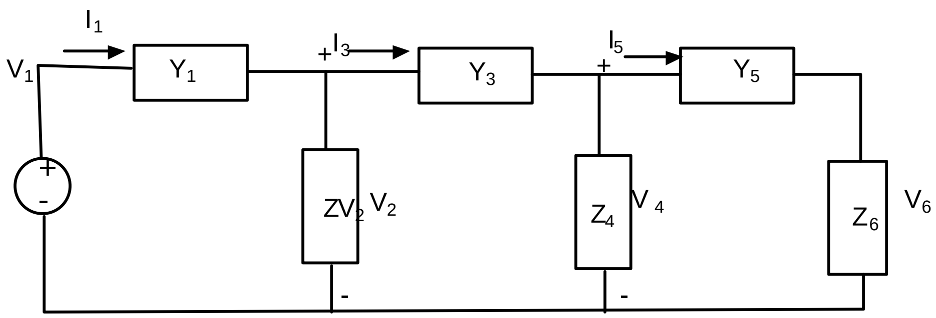


Fig :-1(b)

From figure, (b)

$I_1 = y_1 (V_1 - V_2)$

$V_2 = Z_2 (I_1 - I_3)$

$I_3 = y_3 (V_2 - V_4)$ ____________(i)

$V_4 = Z_4 (I_3 - I_5)$

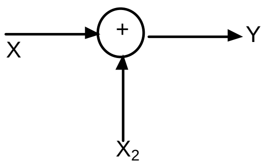
$I_5 = y_5 (V_4 - V_6)$

$V_6 = Z_6 I_5$

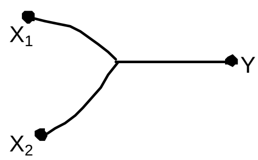
Recall that,

Y = Gx can be represented in the form

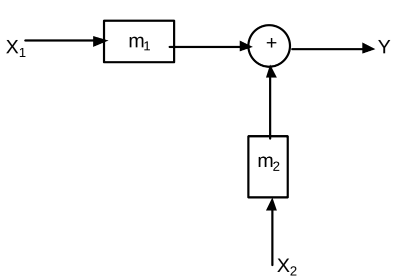
(2) $Y = X_1 + X_2$



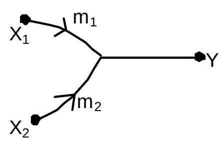
or,



(3) $y = m_1x_1 + m_2x_2$

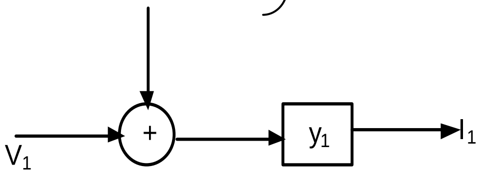


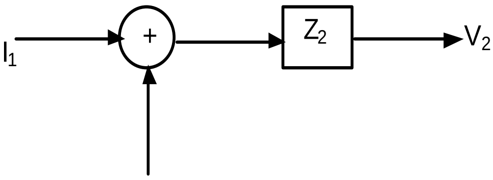
or,

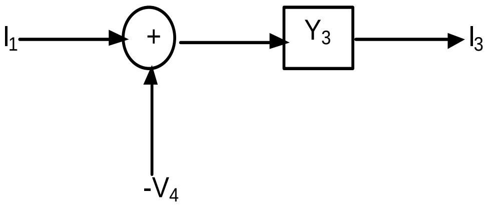


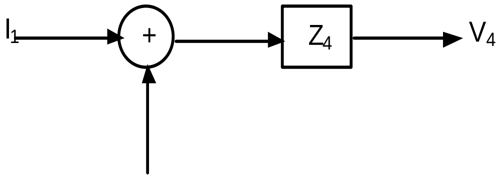
Modifying eq^n (i) set,

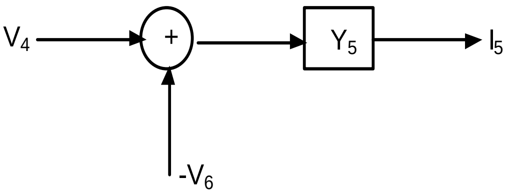
$$\left.\begin{aligned} I_1 &= y_1 (V_1 + (-V_2)) \\ V_2 &= Z_2 (I_1 + (-I_2)) \\ I_3 &= y_3 (V_2 + (-V_4)) \\ V_4 &= Z_4 (I_3 + (-I_5)) \\ I_5 &= y_5 (V_4 + (-V_6)) \\ V_6 &= Z_6 I_5 \end{aligned}\right\} \ldots\ldots\ldots\ldots(ii)$$

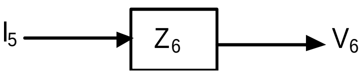


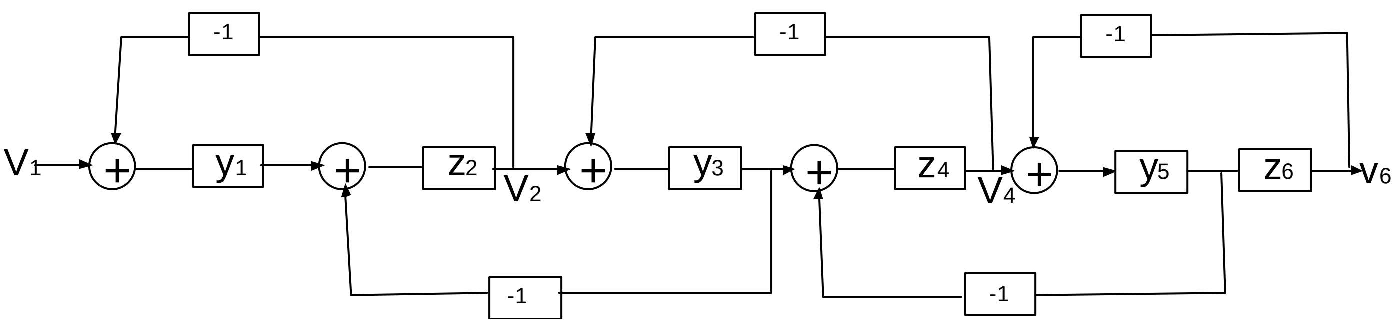












In active ckt, we can not realize the current, so we replace all the currents by their respective Voltages.

i.e, we replace,

$I \rightarrow V_1$

$Y \rightarrow T_y$

$Z \rightarrow T_Z$

From eq^n (iii)

$$\frac{I_1}{y} = \frac{y_1}{y}(v_1 - v_2)$$

or, $V_{T1} = T_{y1}(v_1 - v_2)$

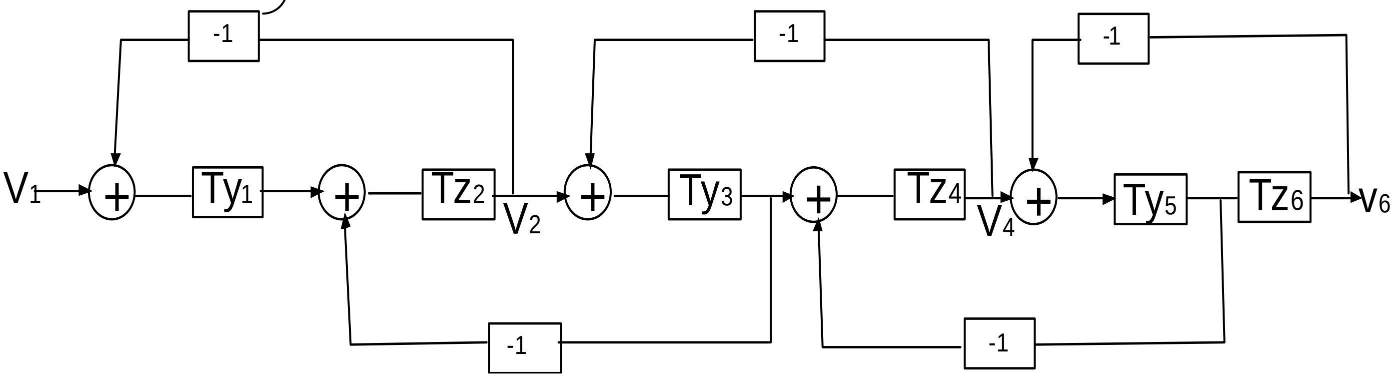
Also, $v_2 = z_2(I_1 - I_3)$

$$= \frac{z_2}{z_1}(I_1 - I_3)z$$

$$v_2 = Tz_2(v_{I1} - v_{I3})$$

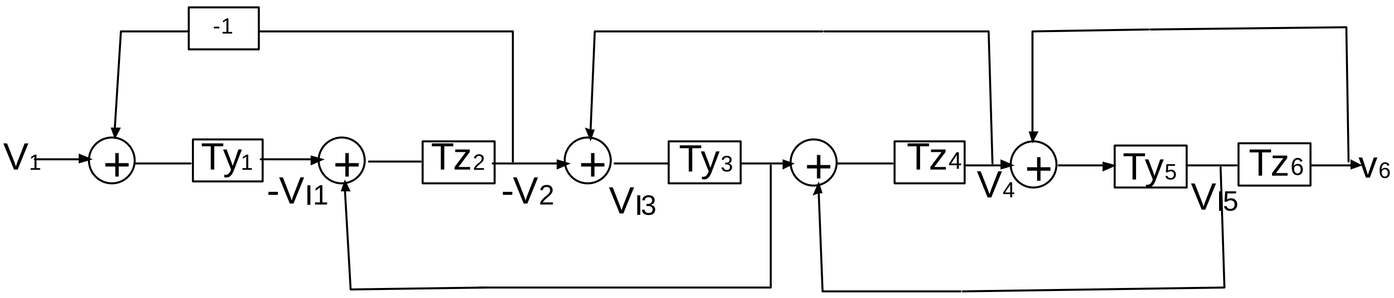
Thus eq^n (ii) can be rewritten as,

$$\left.\begin{aligned} VI_1 &= Ty_1 (V_1 - V_2) \\ V_2 &= TZ_2 (V_{I1} - V_{I3}) \\ VI_3 &= Ty_3 (V_2 - V_4) \\ V_4 &= TZ_4 (V_{I3} - V_{I5}) \\ VI_5 &= Ty_5 (V_4 - V_6) \\ V_6 &= TZ_6 V_{I5} \end{aligned}\right\} \quad \text{..........(iii)}$$

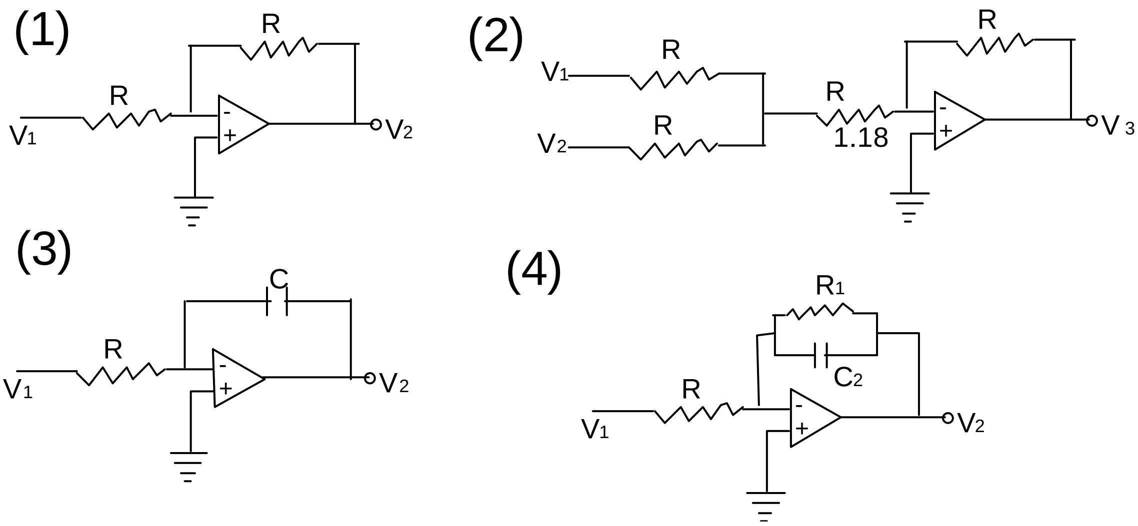


Modifying eq^n set (iii)

$$\left.\begin{aligned} -VI_1 &= -Ty_1 (V_1 - V_2) \\ -V_2 &= TZ_2 (-V_{I1} + V_{I3}) \\ -VI_3 &= -Ty_3 (-V_2 + V_4) \\ V_4 &= TZ_4 (V_{I3} - V_{I5}) \\ -VI_5 &= -Ty_5 (V_4 - V_6) \\ -V_6 &= T_{Z6} (-V_{I5}) \end{aligned}\right\} \text{..........(iv)}$$

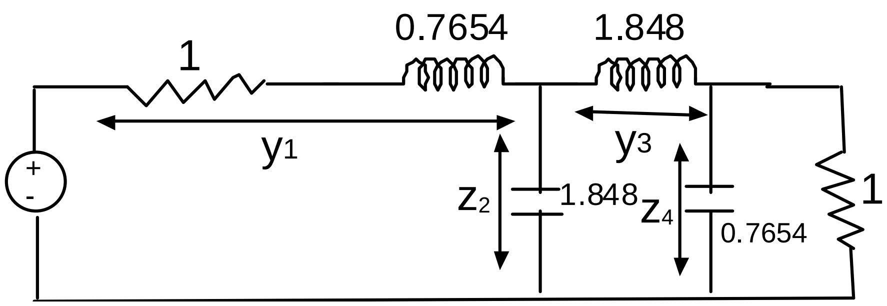


Certain important op-Amp configuration:-

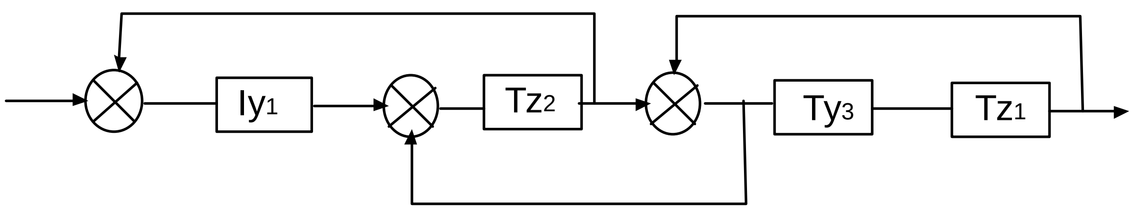


Example:-:01

Design a 4th order low pass Butterworth filter with $\frac{1}{2}$ power frequency of 10^4 rad/sec. The filter must be implemented on leapfrog active filter simulation.



Sol^n:-



From fig (i)

$Z_1 = R_1+L_1S$

$$\therefore y_1 = \frac{1}{R_1 + L_1 S} = \frac{\frac{1}{L_1}}{s + \frac{R_1}{L_1}}$$

$$z_2 = \frac{1}{c_2 s}$$

$$y_3 = \frac{1}{L_3 s}$$

$$y_4 = c_4 + \frac{1}{R_L} \Rightarrow z_4 = \frac{R_L}{R_L c_4 s + 1} = \frac{\frac{1}{c_4}}{s + \frac{1}{c_4 R_L}}$$

$$= \frac{R_L c_4 s + 1}{R_L}$$

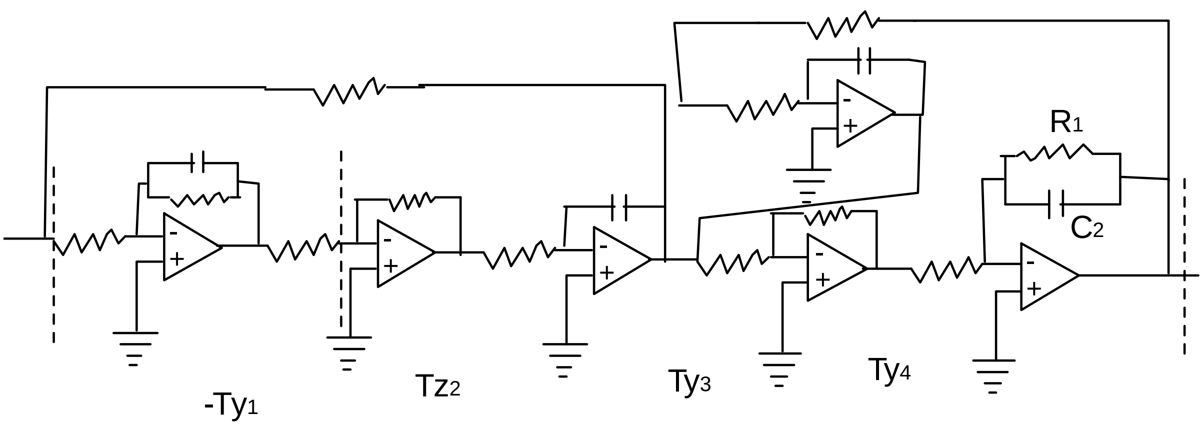
Now,

$$y_1 = \frac{\frac{1}{L_1}}{s + \frac{R_1}{L_1}} \Rightarrow -Ty_1 = \frac{\frac{-1}{L_1}}{s + \frac{R_1}{L_1}}$$

$$z_2 = \frac{1}{c_2 s} \Rightarrow Tz_2 = \frac{1}{c_2 s} = (-1)\left(\frac{-1}{c_2 s}\right)$$

$$y_3 = \frac{1}{L_3 s} \Rightarrow -Ty_3 = -\frac{1}{L_3 s}$$

$$z_4 = \frac{\frac{1}{c_4}}{s + \frac{1}{c_4 R_L}} \Rightarrow Tz_4 = -1\left(\frac{-\frac{1}{c_4}}{s + \frac{1}{c_4 R_L}}\right)$$



Comparison

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$$\Rightarrow -Ty_1 = \frac{-\frac{1}{L_1}}{s+\frac{R_L}{L_1}} \equiv \frac{\frac{1}{R_1c_2}}{s+\frac{1}{R_2c_2}} \qquad [R_1 = R_2 = 1]$$

$$Tz_2 = \frac{1}{c_2 s} \equiv (-1)\left(-\frac{1}{c_2 s}\right) = (-1)\left(-\frac{1}{Rcs}\right) \qquad [R = 1]$$

$$-Ty_3 = -\frac{1}{L_3 s} \equiv \left(-\frac{1}{Rcs}\right) \qquad [R = 1]$$

$$Yz_4 = (-1)\left(\frac{\frac{1}{c_4}}{s+\frac{1}{c_4R_L}}\right) \equiv (-1)\left(\frac{-\frac{1}{R_1c_2}}{s+\frac{1}{R_2c_2}}\right) \qquad [R_1 = R_2 = 1]$$

Now, for frequency and magnitude scaling

W_0 = 1rad/sec

$\Omega_0 = 10^4$rad/sec

$\therefore$ kf = 10^4

Let, km =10^4

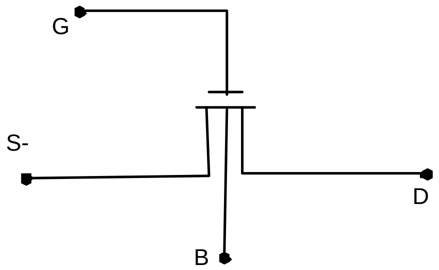
Now scale.

In summary, leapfrog simulation can be done in the following steps.

(1) Choose a suitable low pass prototype which meets the following specifications.(see table 13.1 of van valkenbutg)
(2) Perform freq transformation if necessary.
(3) Identify the various y & z in the form of block diagram.
(4) Select the leapfrog block diagram to simulate the ckt.
(5) Find the active ckt that realize each of the blocks.
(6) Arrange the ckt with necessary components.
(7) Scale the ckt to meet the actual requirements.

Chapter:-12

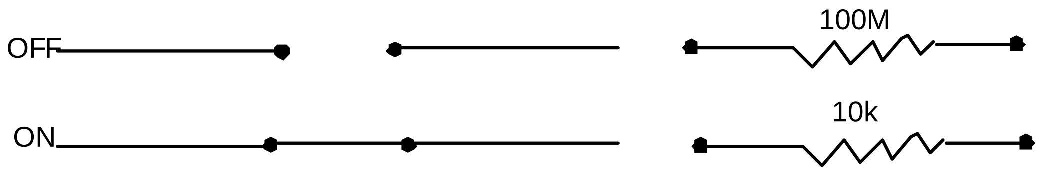
The Mos switch

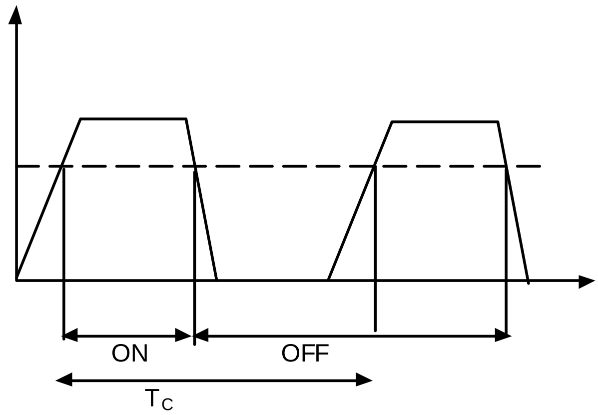


→ Preferred for voice frequency filters

→ If $V_{GS}<V_T$, switch off, behave as open ckt between S & D.

→ If $V_{GS}>V_T$, switch on, behave as short ckt between S & D.





T_C is the time period.

$$fc = \frac{1}{T_C} = \text{Switching frequency}$$

The equivalent representation for such case is

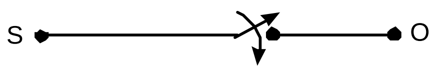
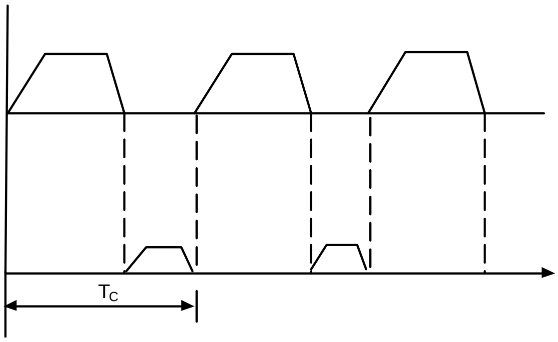


Fig:- SPST Switch

(single pole single throw switch)

Again, consider the two phase clock.



Note that, the frequency do not overlap and when ϕ_1, is OFF, ϕ_2 is ON, and vice versa.

The representation for these cases will be

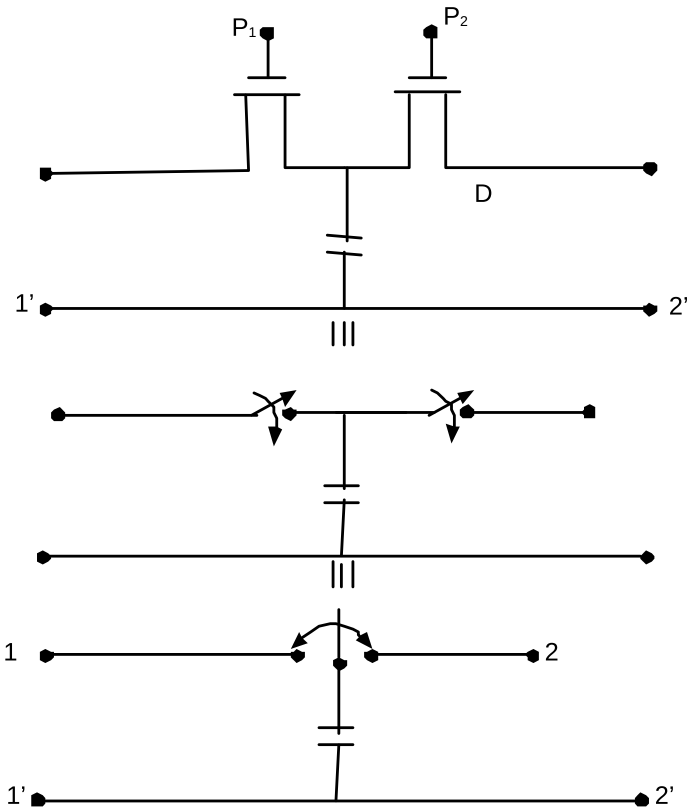
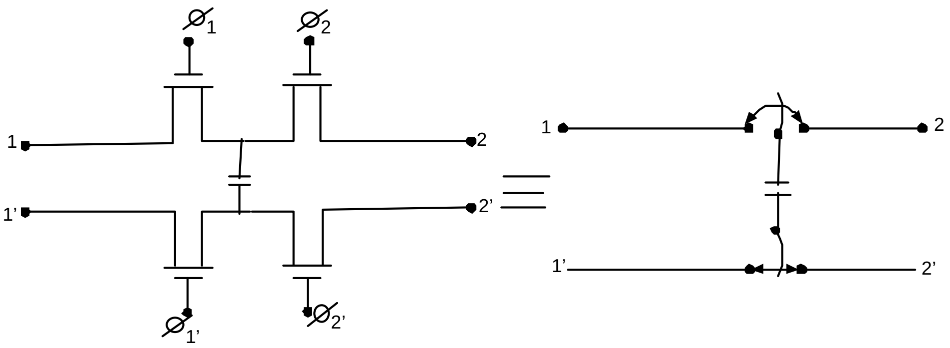
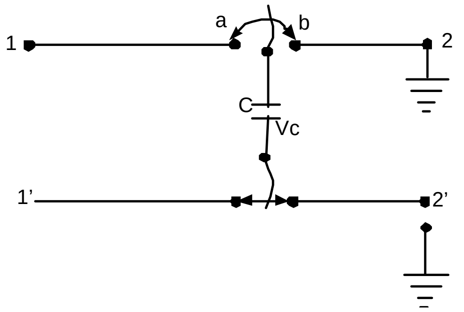


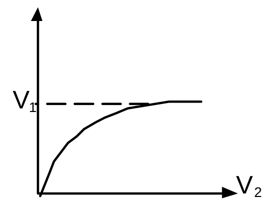
Fig:- SPDT(SIngle pole double through switch)

DPDT (Double pole Double through) switch:-

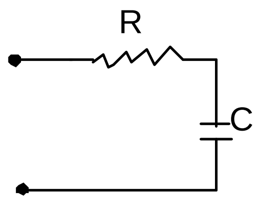


Let us consider the following specification ckt for DPDT switch.





The equivalent ckt for S/W is



When S/W is brought to position b,

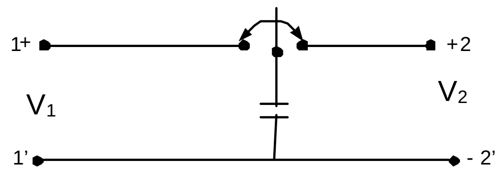
$V_2 = -V_C$

$V_2 = -V_1$

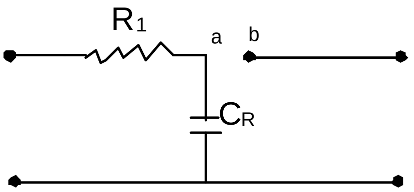
This show that DPDT S/w acts as an inverter

Q**. How can you use DPDT as an Inverter**?

Simulation of resistor by switched capacitor

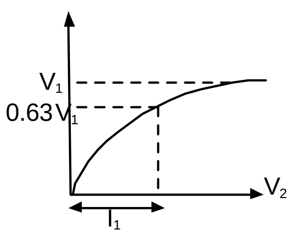


Let us assume SPDT switch as shown in fig (i) let V_1 (t) be the i/p voltage, if s/w is at position 'a' then the eq^n ckt will be:



The capacitor will get charge for

$$\tau_1 = R_1C_R$$



Now let us more the switch to position 'b'.

The charge transferred will be

$$q = C_R (V_1 - V_2)$$

$\therefore$ The current in this is

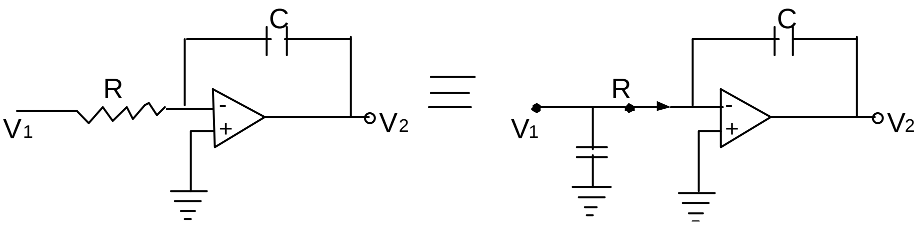
$$i(t)=\frac{q}{t}=C_R(v_1-v_2)$$

$$\frac{C_R(v_1-v_2)}{Tc}=\frac{(v_1-v_2)}{Rc}$$

$$\therefore RC=\frac{1}{fcCR}$$

Switched capacitor for op-amp based angle operation:-

(1) **Integrator**



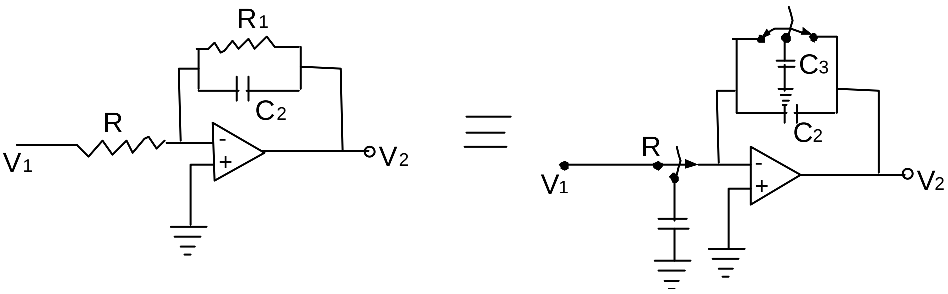
From fig

$$R=\frac{1}{fxC_R}$$

$$\therefore \frac{v_2}{v_1}=-\frac{1}{\frac{1}{fcC_R}.c.s}=-fc.\frac{C_R}{c}.s$$

$$\therefore T(s)=-fc.\frac{CR}{c}.s$$

(2) **loosy Integrator**

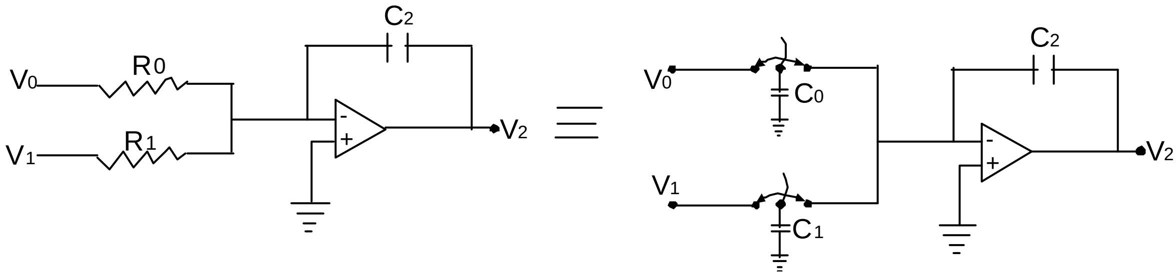


Here,

$$R_1=\frac{1}{fcc_1}\ \&\ R_3=\frac{1}{fcc_3}$$

$$\therefore \frac{v_2}{v_1}=\frac{-fc\frac{c_1}{c_2}}{s+fc\frac{c_3}{c_2}}$$

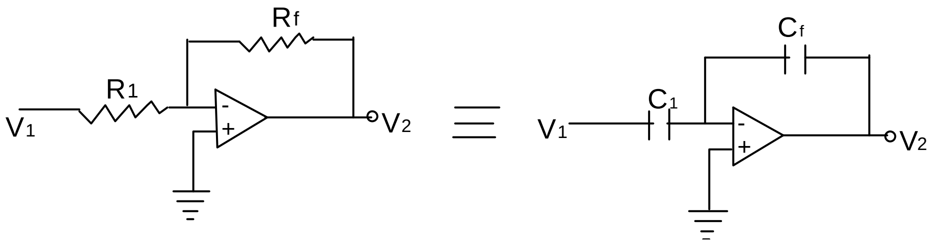
(3) **Adder Integrator**



$$v_2 = \frac{-1}{R_0 c_2 s} v_0 - \frac{1}{R_1 c_2 s}.v_1$$

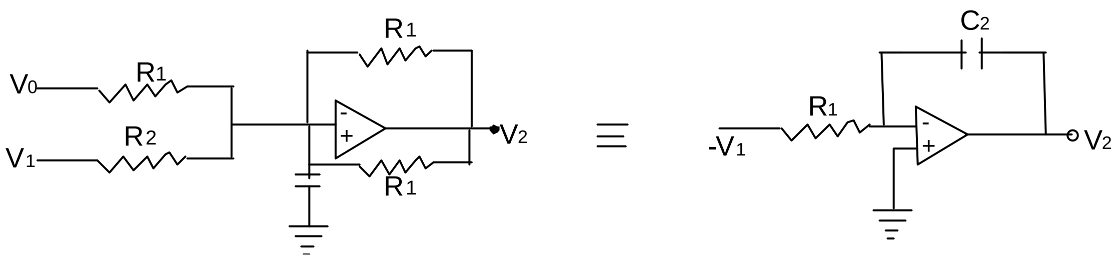
$$= -fc\frac{c_0}{c_2}.\frac{1}{s}.v_0 - fc\frac{c_1}{c_2}.\frac{1}{s}.v_1$$

(4) **Inverting**



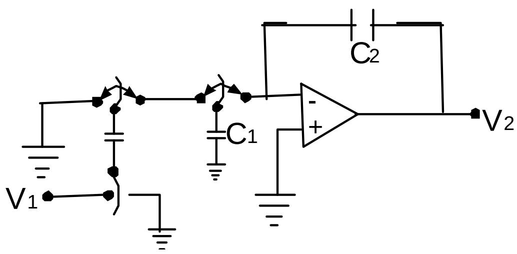
$$\frac{v_2}{v_1} = -\frac{Rf}{R_1} = \frac{-\frac{1}{fccf}}{-\frac{1}{fcc1}} = -\frac{c_1}{cf} \qquad \frac{v_2}{v_1} = \frac{\frac{1}{cfs}}{\frac{1}{c_1 s}} = \frac{c_1}{cf} \qquad \therefore \frac{v_2}{v_1} = -\frac{c_1}{cF}$$

(5)



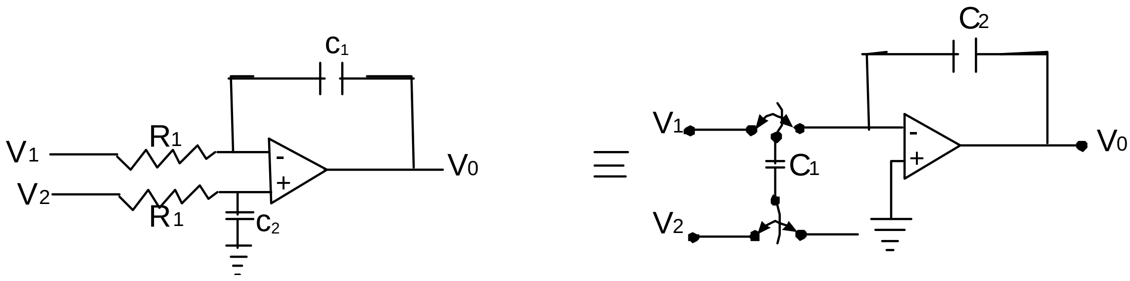
$$\frac{v_2}{v_1} = \frac{1}{R_1 c_2 s} \Rightarrow v_2 = (-v_1).\left(-\frac{1}{R_1 c_2 s}\right)$$

Thus, the ckt using switched capacitor will be,



$$\frac{v_2}{v_1} = fc.\frac{c_1}{c_2}.s$$

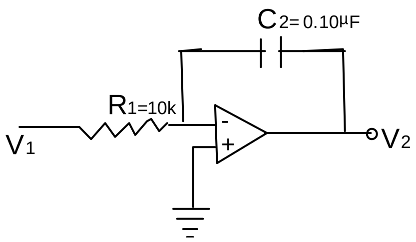
(6)

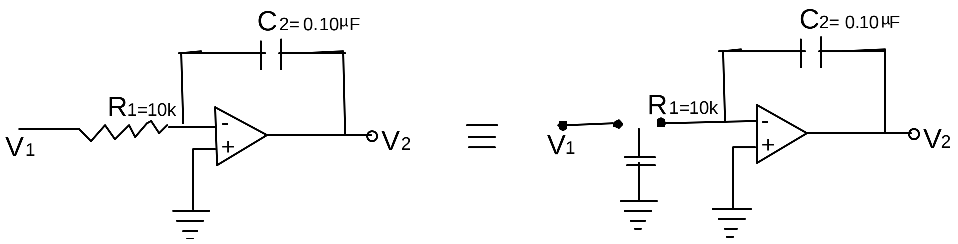


$$\frac{v_2}{v_1} = \frac{1}{R_1 c_2 s}(v_1 - v_2)$$

<u>Example</u>:-01

Realize the given circuit by switched capacitor.





$$\frac{v_2}{v_1} = -\frac{1}{R_1 c_2 s}$$

But, $R_1 = \frac{1}{fcc_1}$

$$\therefore \frac{v_2}{v_1} = -fc.\frac{c_1}{c_2}.\frac{1}{s}$$

$$\Rightarrow \tau = R_1 c_2 = 10\times10^3 \times 0.01\times10^{-6} = 10^{-4}$$

or, $R_1 c_2 = \frac{1}{fc}.\frac{c_1}{c_2} = 10^{-4}$

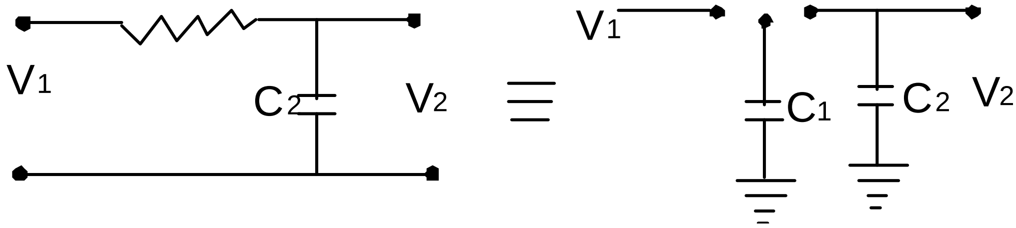
$$\therefore \frac{1}{fc}.\frac{c_1}{c_2}10^{-4}$$

Let, fc = 10 kHz

$$\therefore \frac{1}{10\times10^3}, \frac{c_1}{0.01\times10^{-6}} = 10^{-4}$$

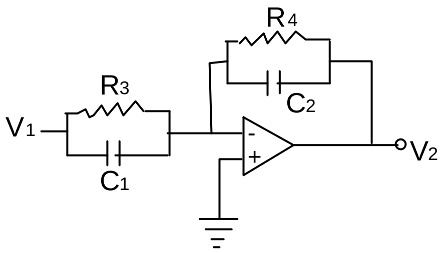
$c_1 = 0.01\mu F$

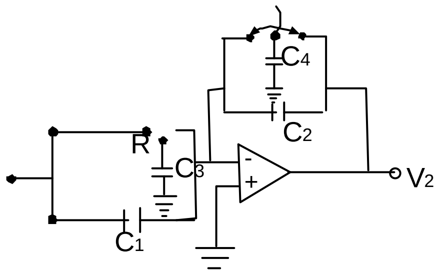
First order filter:-



$$T(s) = \frac{\frac{1}{R_1 c_2}}{s + \frac{1}{R_1 c_2}}$$

$$T(s) = \frac{fc\frac{c_1}{c_2}}{s + fc\frac{c_1}{c_2}}$$





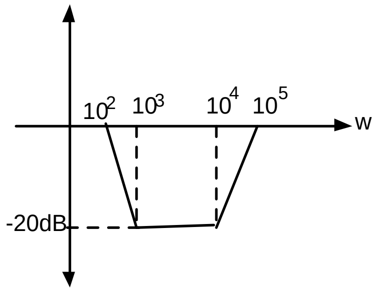
$$T(s) = -\frac{c_1}{c_2}\left(\frac{s + \frac{1}{R_3 c_1}}{s + \frac{1}{R_4 c_2}}\right)$$

$$T(s) = -\frac{c_1}{c_2}\left(\frac{s + fc.\frac{c_3}{c_1}}{s + fc.\frac{c_4}{c_2}}\right)$$

$$\left[\sin ce : R_3 = \frac{1}{fcc_3} \;\&\; R_4 = \frac{1}{fcc_4}\right]$$

Example:- 01

Design a switched capacitor filter from the following plot.



From the plot,

$$T(s) = \frac{(s+a)(s+b)}{(s+c)(s+d)}$$

$$= \frac{(s+10^3)(s+10^4)}{(s+10^2)(s+10^5)}$$

$$= -\left[\left(\frac{(s+10)^3}{s+10^2}\right)\right]\left[-\left(\frac{s+10^4}{s+10^5}\right)\right]$$

$$= T_1(s) \quad T_2(s)$$

for,

$$T1(s) = -\frac{(s+10^3)}{(s+10^2)} \text{……….(i)}$$

Comparing eq^n (i) with

$$T(s) = -\frac{c_1}{c_2}\left(\frac{s + fc\frac{c_4}{c_1}}{s + fc\frac{c_4}{c_1}}\right)$$

Take, fc = 10 kHz

$C_1 = C_2$ =10 pf

$\therefore$ C3 = 1 pf

$\therefore$ C4 = 0.1pf

For,

$$T_2(s) = \frac{-(s+10^4)}{(s+10^5)}$$

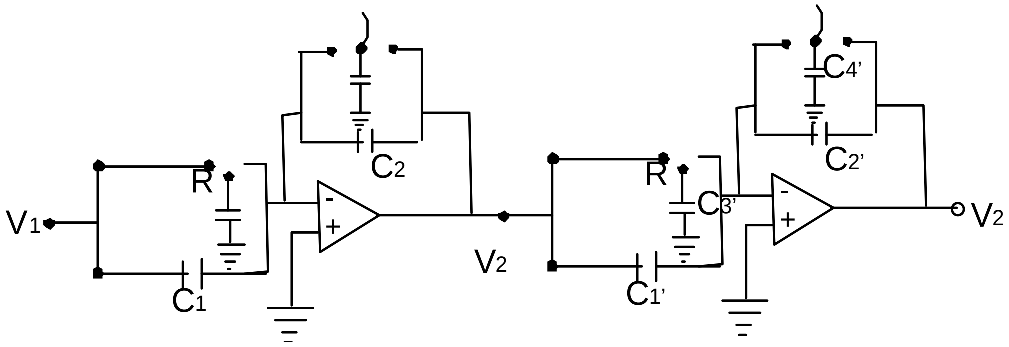
Comparing,

$$T(s) = \frac{-c_1}{c_2}\left(\frac{s + fc\frac{c_3}{c_1}}{s + fc\frac{c_4}{c_1}}\right)$$

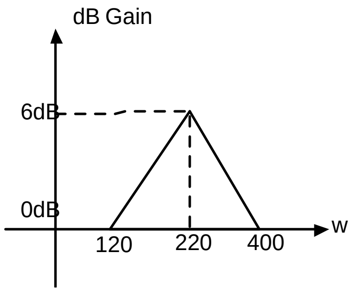
$C_1 = C_2$ =10 pf

$\therefore C_3$ = 10 pf

C_4 = 100 pf



Example:-02



$$T(s) = \frac{(s+100)(s+400)}{(s+200)^2}$$